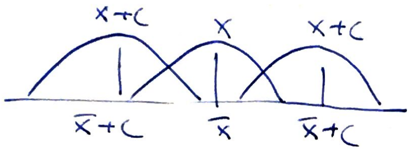
5/22/19 Section 8 LB

Covariance and Correlation properties

X: random variable

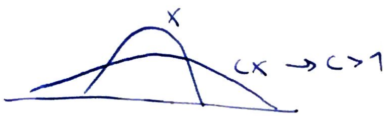
c: constant

1) $Var(X + c) = Var(X)$

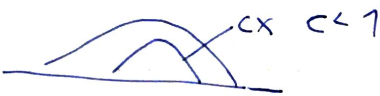


Spread stays the same

2) $Var(cX) = c^2 Var(X)$



Spread increases



3) X and Y random variables

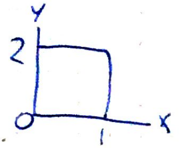
$Var(X+Y) = Var(X) + Var(Y) + 2Cov(X,Y)$

If X and Y are independent then $Cov(X,Y) = 0$

and $V(X+Y) = Var(X) + Var(Y)$

$\boxed{\text{Problem 12 pg. 255}}$

PDF of X, Y: $f(x,y) = \begin{cases} \frac{1}{3}(x+y) & 0 \le x \le 1 \\ & 0 \le y \le 2 \\ 0 & \text{else} \end{cases}$



$0 \le x \le 1$

$0 \le y \le 2$

$\boxed{\text{Cov}(aX, bY) = a \cdot b \cdot \text{Cov}(X, Y) \quad \text{one more property}}$

$Var(2x - 3y + 8)$

We can drop the constant 8 because it doesn't have a random variable

$= Var(2x - 3y)$

$= Var(2x + (-3)y)$

$= Var(2x) + Var(-3y) + 2\,Cov(2x, -3y)$

$= 2^2 Var(x) + (-3)^2 Var(y) + 2(2)(-3) Cov(x, y)$

$= 4\,Var(X) + 9\,Var(Y) - 12\,Cov(X, Y)$

$Var(X) = E(x^2) - [E(x)]^2$

$Cov(X, Y) = E(X, Y) - E(X)E(Y)$

Marginal distribution of X

$$f_X(x) = \int_0^2 \frac{1}{3}(x+y)\,dy = \frac{1}{3}\left(xy + \frac{y^2}{2}\right)\Big|_0^2 = \frac{1}{3}(2x + 2)$$

$$= \begin{cases} \frac{2}{3}(x+1) & \text{for } 0 < x < 1 \\ 0 & \text{else} \end{cases}$$

$$E(x) = \int_0^1 x \cdot \frac{2}{3}(x+1)\,dx = \frac{2}{3}\left(\frac{x^3}{3} + \frac{x^2}{2}\right)\Big|_0^1 = \frac{5}{9}$$

$Cov(X,Y) = \frac{2}{3} - \left(\frac{5}{9}\right)\left(\frac{11}{9}\right) = -\frac{1}{81}$

$Var(2x-3y+8) = 4\left(\frac{13}{162}\right) + 9\left(\frac{23}{81}\right) - 12\left(-\frac{1}{81}\right)$

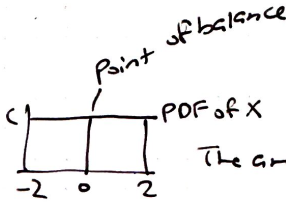
$= \boxed{\frac{245}{81}}$ Final Answer

Problem 3 pg. 255

Clearly X and Y aren't independent because $Y = X^6$ so Y obviously depends on X.

X, Y random variables

$X \sim Uniform(-2, 2) \rightarrow$



The area must equal 1

$Y = X^6$

$\rho(X,Y) = 0$??

$$\rho(X,Y) = \frac{Cov(X,Y)}{\sqrt{Var(X) \cdot Var(Y)}}$$

We need to prove: $Cov(X,Y) = 0$

$$Cov(X,Y) = Cov(X, X^6) = E(X \cdot X^6) - E(X)E(X^6)$$

$$= E(X^7) - E(X)E(X^6)$$

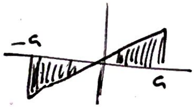
To find c:

Area rectangle $= 1 = c \cdot 4$ so $c = \frac{1}{4}$

PDF of X: $f_X(x) = \begin{cases} \frac{1}{4} & \text{for } -2 \leq x \leq 2 \\ 0 & \text{else} \end{cases}$

Intuitively we can say that the expected value $E(X) = 0$

$$\int_{-a}^{a} f(x)\,dx = 0 \quad \text{when } f(x) \text{ is odd}$$



The positive and negative areas cancel out to zero

$$E(x^7) = \int_{-2}^{2} \frac{x^7}{4}\,dx = 0$$

odd function

$$Cov(X, Y) = E(x^7) - E(X)E(x^6) = 0 - 0(E(x^6)) = 0$$