4/4/2019 Lecture 2

Office Hours start on Friday 4/15/19 (Tomorrow)

Monday
- 9-10am - LB - BE room 312C/D
- 12-1pm - XY - BE room 312C/D

Tuesday
- 9-10 am - RG - E2 room 586
- 3:30-5pm - OD - JL

Wednesday
- 8-9am - LB - BE room 312C/D

Thursday
- 9-10am - RG - E2 room 586
- 3:30-5pm - OD - JL

Friday
- 9:30-10:30am - XY - BE room 360

Contacting thru Email
Put "AMS 13" in the subject title so he doesn't miss it

There may be late assignments but the specifics about how that will work are still to be determined

HW can be photoscans or typed. Make sure it's legible so pencil would not be best.

Case Study 1

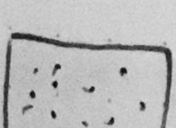
P(1 or more T-S in 5 children, both parents carriers) = ?

set

Probability of → P(A) or

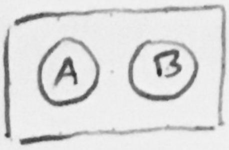
true/false proposition

In a Venn Diagram, the box represents the set of possibilities, which are equally likely.

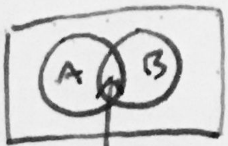
 $P(\square) = 1 = 100\%$

$P(A) = \frac{A}{\square}$ (1)

Addition Rule for OR



$P(A \text{ or } B) = P(A) + P(B)$



$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

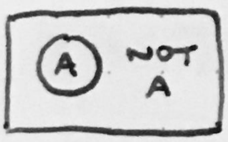
intersection / overlap / A and B

takes care of double count

$0 \leq P(A) \leq 1$

impossibility certainty

When A and B have no overlap then they are <u>mutually exclusive</u>.



$P(A \text{ or } (\text{not } A)) = P(A) + P(\text{not } A) = 1$

$P(A) = 1 - P(\text{not } A)$ ← indirect direct

$P(A \text{ and } B) \overset{?}{=} P(A) \text{ ? } P(B)$

2 cases

Population sampling

$n=3 \begin{bmatrix} 1 \\ 2 \\ 9 \end{bmatrix} \xrightarrow{\text{at random}} \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} n=2$

$P(Y_1 = 9 \text{ and } Y_2 = 9) = ? = \frac{1}{9}$ $P(Y_1 = 9) = \frac{3}{9} = \frac{1}{3}$

IID (at random w/ replacement)

ELM? Yes 9 elemental outcomes

2nd draw

	1	2	9
1	(1,1)	(1,2)	(1,9)
2	(2,1)	(2,2)	(2,9)
1st draw 9	(9,1)	(9,2)	(9,9)

$P(Y_2 = 9) = \frac{3}{9} = \frac{1}{3}$

$P(Y_1 = 9 \text{ and } Y_2 = 9) = P(Y_1 = 9) \times P(Y_2 = 9) = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$

SRS (at random w/out replacement)

2nd draw

1st draw	1	2	9
1	~~(1,1)~~	(1,2)	(1,9)
2	(2,1)	~~(2,2)~~	(2,9)
9	(9,1)	(9,2)	~~(9,9)~~

ELM? = Yes

$P(Y_1 = 9 \text{ and } Y_2 = 9) = 0$

$P(Y_1 = 9) = \frac{2}{6} = \frac{1}{3}$

$P(Y_2 = 9) = \frac{2}{6} = \frac{1}{3}$

$P(Y_1 = 9 \text{ and } Y_2 = 9) = 0 \neq \frac{1}{3} \cdot \frac{1}{3} = P(Y_1 = 9) \cdot P(Y_2 = 9)$

Conditional probability: The probability of one thing affects the probability of another

$P(A \mid B)$ = Probability of A given B

$= \dfrac{\text{A and B intersection}}{\text{B}}$

$$P(A \mid B) = \begin{cases} \dfrac{P(A \text{ and } B)}{P(B)} & \text{if } P(B) > 0 \\ \text{undefined} & \text{if } P(B) = 0 \end{cases}$$

$P(A \text{ and } B) = P(B) \times P(A \mid B)$ General Product Rule for AND

$P(B \mid A) = \dfrac{P(B \text{ and } A)}{P(A)} = \dfrac{P(A \text{ and } B)}{P(A)}$

$P(A \text{ and } B) = P(A) \times P(B \mid A)$ Chain Rule for AND

$P(\bar{Y}_1 = 9 \text{ and } \bar{Y}_2 = 9) = P(\bar{Y}_1 = 9) \times P(\bar{Y}_2 = 9 \mid \bar{Y}_1 = 9)$

$= \frac{1}{3} \times 0 = 0$ ✓

$P_{IID}(\bar{Y}_1 = 9 \text{ and } \bar{Y}_2 = 9) = P_{IID}(\bar{Y}_1 = 9) \times P_{IID}(\bar{Y}_2 = 9 \mid \bar{Y}_1 = 9)$

$= P_{IID}(\bar{Y}_1 = 9) \times P_{IID}(\bar{Y}_2 = 9)$

Bayesian

If info about A doesn't change the chances about B and vice versa then A and B are independent of each other.

Frequentist

A and B are independent iff (if and only if)
$P(A \text{ and } B) = P(A) \times P(B)$

I = Independent
I = Identically
D = Distributed

T-S Case

P(1 or more T-S babies) = P(*)

\# of T-S babies: 0, 1, 2, 3, 4, 5 = 6 EOS

if ELM applies then $P(*) = \frac{5}{6}$

not realistic argument

if ELM doesn't apply then $P(1) > P(0)$

P(*) = P(exactly 1 T-S or.... or exactly 5 T-S)

= P(exactly 1) + ... + P(exactly 5)

P(*) = 1 − P(zero T-S babies)

= 1 − P(not T-S on 1st child and not T-S on 2nd child and... and not T-S on 5th child)

Biology → IID

$$P(*) = 1 - P\binom{\text{not T-S}}{\text{on 1st}} \times P\binom{\text{not T-S}}{\text{on 2nd}} \times \dots \times P\binom{\text{not T-S}}{\text{on 5th}}$$

$$= 1 - (1 - \tfrac{1}{4}) \times (1 - \tfrac{1}{4}) \times \dots \times (1 - \tfrac{1}{4})$$

$$= 1 - (1 - \tfrac{1}{4})^5$$

= 76% chance of having T-S babies

This was a calculation with Binomial Distribution.

<u>Def</u>: An experiment $\mathcal{E}$ is a data-generating process in which all possible outcomes can be listed before $\mathcal{E}$ is performed.

<u>Def</u>: An event E is a set of possible outcomes of an experiment $\mathcal{E}$

Ex: T-S Disease where $\mathcal{E}$ = the process by which the husband & wife end up w/ 5 children, each a T-S baby or not. The E of interest is $E = \{\geq 1 \text{ T-S baby}\}$

<u>Def</u>: The sample space S is the set of all possible outcomes of $\mathcal{E}$

N = not T-S   T = is T-S $S = \{NNNN\dots TTTTT\}$

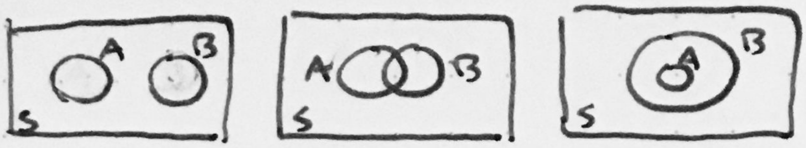
Each of the 5 babies is T or N so $S = 2^5 = 72$

S is an an example of a product space:

$$\underbrace{\{T, N\} \times \{T, N\} \times \dots \times \{T, N\}}_{5} = \{T, N\}^5$$

<u>Def</u>: The complement A^c of a set A contained in S consists in all the elements of S not in A

$s \in S$ means that $\left\{\begin{matrix}\text{outcome} \\ \text{or} \\ \text{element}\end{matrix}\right\}$ s belongs to S



← A is contained in B ($A \subset B$)

If A, B, C are events then

(a) $A \subset B$ and $B \subset A \longleftrightarrow A = B$

(b) $A \subset B$ and $B \subset C \longrightarrow A \subset C$ (← Consequences)

If A and B are events, $A \subset B$ iff A occurs then so does B

<u>Def</u>: The cardinality of a set A (written $|A|$) is the # of distinct elements in A.

Ex: T-S Case : $|S| = 32$ because there 32 ways to have the babies

<u>Def</u>: The set of all subsets of a given set is called the <u>power set</u> of S, denoted by 2^S

If $|S| = n$, then $|2^S| = 2^n$

In other words, if S has n distinct elements then there are 2^n distinct subsets of S.

<u>Def</u>: A set that has no elements in it is an empty set ($\emptyset$)

Ex: If $S = \{a, b, c\}$ then $|S| = 3$ and the power set has $2^3 = 8$ sets in it.

$\{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, \{a, b, c\}\}$ = Power set

History:
Given any set sample space S, Kolmogorov (1933) wanted to define probs. in a logically-internally-consistent manner (aka free from contradictions) to all of the sets in 2^S.

If $|S|$ is finite, it turns out that nothing nasty can happen. But if $|S|$ is infinite, nasty things can happen.

<u>Def</u>: A set w/ an infite # of distinct elements is called an infinite set