4/11/19

Thursday, April 11, 2019 1:30 PM

This, next time: (see sylabus) reading: see syllabus

Rev. T. Bayes (1760)

village

some people dying (data)

possible causes
- bad water
- bad air
- bad food
- disease(s)

⋮

determenestic causality:

probabilistic causality

cause → effect

? U D

unknown data

P(cause | effect) = ? hard

P(effect | cause) = easy (easier)

$P(U|D) = ?$ $P(D|U)$ $P(D) > 0$

$P(U|D) = \frac{P(U \text{ and } D)}{P(D)} \rightarrow P(U \text{ and } D) = P(D) \cdot P(U|D)$

$\rightarrow P(D|U) = \frac{P(D \text{ and } U)}{P(U)} \rightarrow P(D \text{ and } U) = P(U) \cdot P(D|U)$

therefore

$$P(D) \cdot P(U|D) = P(U) \cdot P(D|U)$$

$$P(U|D) = \frac{P(U) \cdot P(D|U)}{P(D)}$$

$$P(\text{unknown} \mid \text{data}) = \frac{P(\text{unknown}) \cdot P(\text{data} \mid \text{unknown})}{P(\text{data})}$$

Bayes's Theorem for T/F propositions

T/F

$$P(U|D) = \frac{P(U) \cdot P(D|U)}{P(D)}$$

$$P(\text{not } U|D) = \frac{P(\text{not } U) \cdot P(D|\text{not } U)}{P(D)}$$

$$\Rightarrow \left[\frac{P(U|D)}{P(\text{not } U|D)}\right] = \left[\frac{P(U)}{P(\text{not } U)}\right] \cdot \left[\frac{P(D|U)}{P(D|\text{not } U)}\right]$$

posterior odds ratio given D

prior odds ratio in favor of U

Bayes $\boxed{+}$ factor

$P(H) = p$

$P(T) = 1 - p = P(\text{not } H)$

odds ratio against H $= \frac{1-p}{p}$

$\frac{P(H)}{P(\text{not } H)} = \frac{p}{1-p} =$ odds ratio in favor of H

→ time

a priori before info | (now) data D (info) | a posterior after info

$O = \frac{p}{1-p} \longleftrightarrow p = \frac{O}{1+O}$

$\boxed{\text{Case Study}}$: The Rasmussen Report aka "WASH 1400" aka "The reactor Safety Study"

problem: estimate P (catastrophic accident at nuclear power plant)

→ simpler events connected with and, or

they assumed independence ⇒ huge underestimate

$$\boxed{P\left(\bigcap_{i=n}^{n} A_i\right) \geq 1 - \sum_{i=1}^{n} P(A_i^c)}$$

→ useful in statistics

$n(n-1) \cdots 1 = n!$ read "n factorial"