5/14/19

Tuesday, May 14, 2019 1:34 PM

This time:

Next time:

chain rule

PDF of Y is $f_Y(y) = \begin{cases} f_X[h^{-1}(y)]\left|\frac{dh^{-1}(y)}{dy}\right| & \text{for } \alpha < y < \beta \\ 0 & \text{else} \end{cases}$

→ continuous $(m=1)$

n rvs $X_1, \dots, X_n$, continuous joint dist with joint PDF $f_{\underset{\sim}{X}}(\underset{\sim}{x})$.

$Y = h(\underset{\sim}{X})$

Univariate (real)

For each real y define $A_y = \{\underset{\sim}{x} : h(x) = y\}$

→ Then PDF of Y is $f_Y(y) = \int \cdots \int_{A_x} f_{\underset{\sim}{X}}(\underset{\sim}{x})\, d\underset{\sim}{x}$.

$Y_{(1)} \triangleq \min(X_1, \dots, X_n)$

$Y_{(n)} \triangleq \max(X_1, \dots, X_n)$

(Take-Home test 2 problem 4)

$F_{Y_{(n)}}(t) = P(Y_{(n)} \le t)$

$\Updownarrow$ iff

$= P(X_1 \le t, X_2 \le t, \dots, X_n \le t)$

$\overset{\perp\!\!\!\perp}{=} P(X_1 \le t) \cdots P(X_n \le t)$

$\overset{IID}{=} [F_X(t)]^n$

lets work out the joint PDF

$(Y_1, Y_2) \triangleq \left(\frac{X_1}{X_2}, X_1 \cdot X_2\right)$

$y_1 = h_1(x_1, x_2) = \frac{x_1}{x_2}$

$y_2 = h_2(x_1, x_2) = x_1 x_2$

$h_1^{-1}(y_1, y_2) = \sqrt{y_1 y_2}$

$h_2^{-1}(y_1, y_2) = \sqrt{\frac{y_2}{y_1}}$

A useful trick

Step 1 Find $Y_2 = h_2(X_1, X_2)$ s.t. it is 1-1 and differentiable

Step 2 work out the joint dist. of (Y_1, Y_2)

Step 3 Integrate Y_2 out for marginal

back to Tay-Sachs

$E(Y) \triangleq \sum_{\text{all } y}^{n}$

So: for an $n \ge 1$ (interger) and $0 < p < 1$,

$Y \sim \text{Binomial}(n, p) \rightarrow E(Y) = n \cdot p$

continuous

PDF of X → $E(X) \triangleq \int_{-\infty}^{\infty} x f_X(x)\, dx$

method 1:

$E(Y) = \int_{\mathbb{R}} y f_Y(y)\, dy$

method 2: (faster)

$E(Y) = \int_{\mathbb{R}} h(x) f_X(x)\, dx$.

conjecture:

$E(X + c) = E(X) + c$

in general $E(aX + b) = aE(X) + b$

$E(X_1 + X_2) \overset{\checkmark}{\overset{?}{=}} E(X_1) + E(X_2)$

Def: A function $g: \mathbb{R}^2 \rightarrow \mathbb{R}$