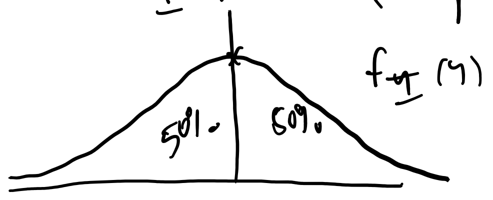
5/2/19

Thursday, May 2, 2019 1:32 PM

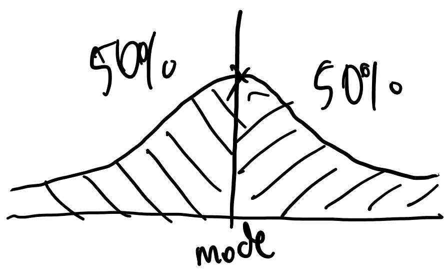
This time: inverse CDFs
Next time: joint distribution

General definition: Y rv with CDF $F_Y(y)$

The $F^{-1}_Y(p)$ is the pth quantile



The 0.5 quantile or the 50th percentile is the median



mode
= point of symmetry
= mean
= median

Measure of spread for the distribution

$F^{-1}_Y(.75) - F^{-1}_Y(.25) =$ interquartile range (IQR)

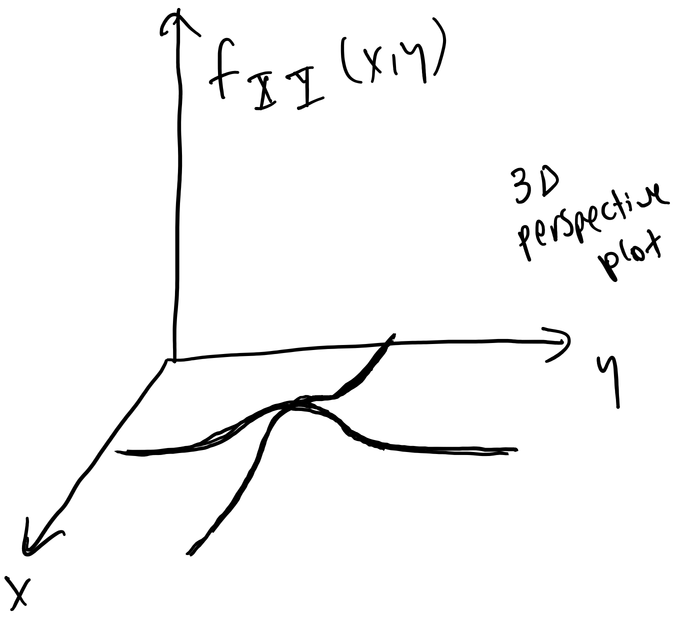
Example 1 $Y \sim \text{Uniform}(a,b)$; then

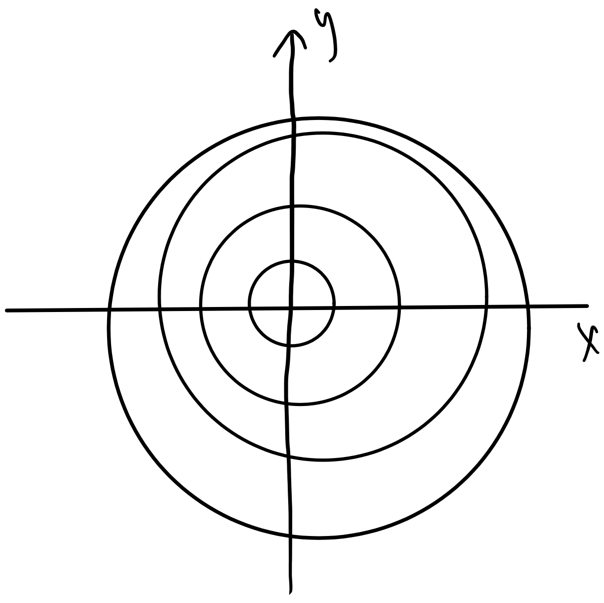
$$F_Y(y) = \begin{cases} 0 & \text{for } y \le a \\ \frac{y-a}{b-a} & a \le y \le b \\ 1 & y \ge b \end{cases}$$

Def. X, Y rvs: the joint (or bivariate) distribution of (X,Y) is the collection $P[(X,Y) \in C]$ of all probabilities for all sets $C \in \mathbb{R}^2$ such that $(X,Y) \in C$ isn't weird.

Consequences (1) discrete $\sum_{\text{all }(x,y)} f_{XY}(x,y) = 1$

(2) For any set C of ordered pairs (x,y) $P[(X,Y) \in C] = \sum_{(x,y) \in C} f_{XY}(x,y)$





2D contour plot

$$\int_0^1 \int_{-\sqrt{y}}^{\sqrt{y}} x^2 y \, dx \, dy \quad \text{or} \quad \int_{-1}^1 \int_{x^2}^1 x^2 y \, dy \, dx$$

$= \frac{4}{21}$ hence $c = \frac{21}{4}$

$$P(X \ge Y) = \iint_{\mathcal{N}} f_{XY}(x,y) \, dy \, dx$$

$$\int_0^1 \left[\int_{x^2}^{x} \frac{21}{4} x^2 y \, dy \right] dx = \frac{3}{20}$$