5/21/19

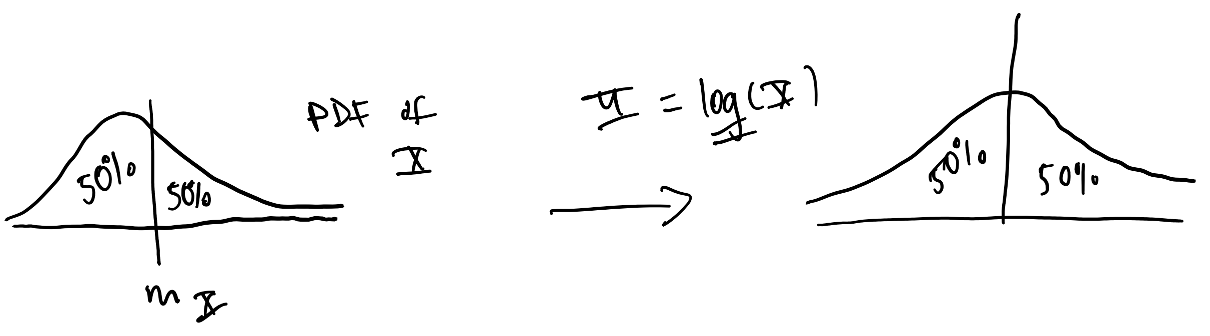
Tuesday, May 21, 2019 1:45 PM

$E(X^k) = \frac{k!}{\lambda^k}$

Example:

$\psi_{S_i}(t) = E(e^{tS_i}) = e^{t1} \cdot p(S_i=1) + e^{t0} p(S_i=0) = [pe^t + (1-p)]$

characteristic function: $\phi_X(t) = E(e^{itX})$



bias $(\hat{Y}) \triangleq E(\hat{X} - X)$

Def: $E[(\hat{X}-X)^2]$ is called the mean squared error

$RMSE = \sqrt{MSE} = \sqrt{E(\hat{X}-X)^2}$

$MSE(\hat{X}) = V(X)$

Def: $E|\hat{X} - X|$ is called mean absolute error (MAE) of $\hat{X}$ as a prediction for X.

Covariance & Correlation

covariance: $C(X,Y) = E[(X-\mu_X)(Y-\mu_Y)]$

$\hookrightarrow C(X,Y) = E(XY) - \mu_X\mu_Y$

$C(X,X) = E[(X-\mu_X)^2] = V(X)$

$C(aX+b, Y) = a\,C(X,Y)$

Def: The process of converting a rv X to standard units (SU) is achieved with the linear transformation $X' = \frac{X - E(X)}{SD(X)}$

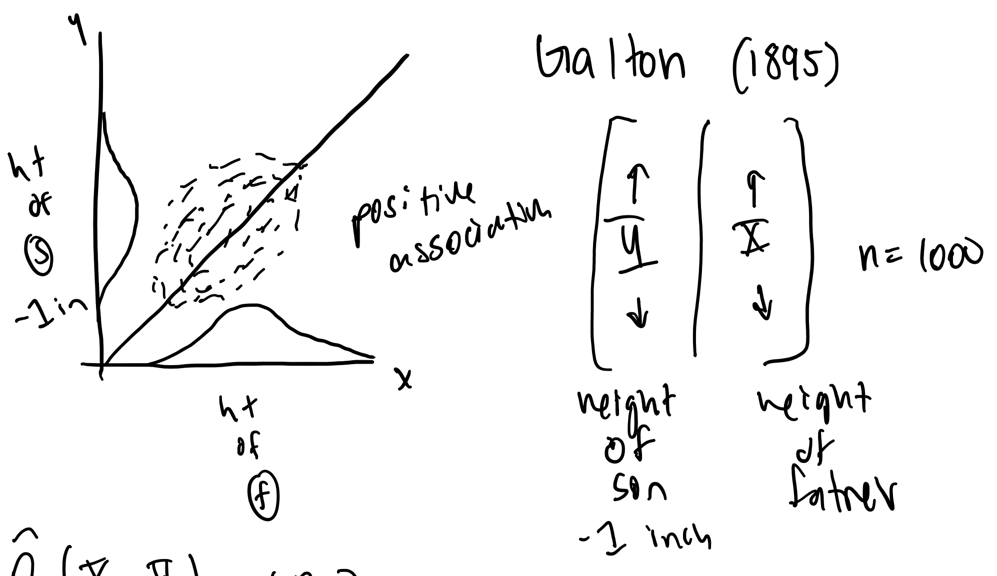
$= \frac{X - \mu_X}{\sigma_X}$

Def. correlation

$\rho(X,Y) = E\left[\left(\frac{X-\mu_X}{\sigma_X}\right)\left(\frac{Y-\mu_Y}{\sigma_Y}\right)\right]$

$= \frac{C(X,Y)}{\sigma_X \sigma_Y}$

$\rho(aX+b, cY+d) = \rho(X,Y)$



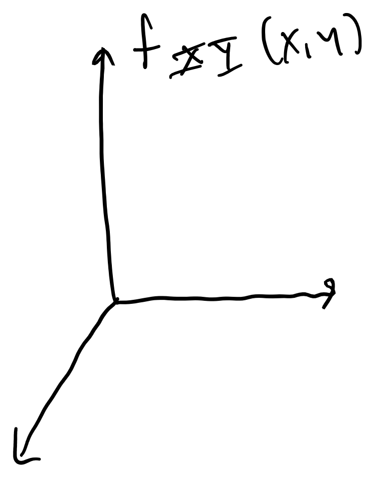
$\hat{\rho}(X,Y) = +0.7$

① Cauchy-Schwarz $\Rightarrow [C(X,Y)]^2 \le \sigma_X \cdot \sigma_Y$

$\hookrightarrow -1 \le \rho(X,Y) \le 1$

② X, Y independent rv $\Rightarrow \rho(X,Y) = 0$

bivariate PDF of (X,Y)



$V(X+Y) = V(X) + V(Y) + 2C(X,Y)$

$V(X+Y) = V(X) + V(Y) - 2C(X,Y)$

Conditional Expectation

- regression line for predicting y from x

continuous

$E(Y|x) = \int_{\mathbb{R}} y\, f_{Y|X}(y|x)\,dy$

discrete

$E(Y|X) = \sum_{\mathbb{R}} y\, f_{Y|X}(y|x)\,dy$

def

$h(x) \triangleq E(Y|X=x)$ then the rv $E(Y|X) \triangleq$