4/23/19

Tuesday, April 23, 2019 1:31 PM

(See syllabus)

$$P(A|B) = \begin{cases} \frac{P(A \wedge B)}{P(B)} & \text{if } P(B) > 0 \\ \text{undefined} & \text{if } P(B) = 0 \end{cases}$$

$$P(A) = \sum_{j=1}^{K} P(B_j)\, P(A|B_j)$$ this is the **Law of Total Probability**

$$P(A|C) = \sum_{j=1}^{K} P(B_j|C)\, P(A|B_j \cap C)$$

Events A, B are independent iff $P(A \wedge B) = P(A) \cdot P(B)$

is equivalent to

$$\left.\begin{aligned} P(A|B) &= P(A) \\ \text{and } P(B|A) &= P(B) \end{aligned}\right\} \text{(Bayesian)}$$

↳ A, B independent iff information about A doesn't change probability of B.

$$P(B_i|A) = \frac{P(B_i)P(A|B_i)}{P(A)} = \frac{P(B_i)\,P(A|B_i)}{\sum_{j=1}^{K} P(B_j) \cdot P(A|B_j)}$$ (how likely data A would be given B_i)

↑ normalizing constant

$$\left(\begin{matrix}\text{posterior} \\ \text{information}\end{matrix}\right) = \frac{\left(\begin{matrix}\text{prior} \\ \text{information}\end{matrix}\right) \cdot \left(\begin{matrix}\text{likelihood} \\ \text{information}\end{matrix}\right)}{\left(\begin{matrix}\text{normalizing} \\ \text{constant}\end{matrix}\right)}$$

Back to Credit Card Screening Case Study

TRUTH	SYSTEM SAYS
B = (card is really bad)	$\oplus$ = (system says bad)
G = (card is really good)	$\ominus$ = (system says good)

We know:

$P(B) = 0.01$ (prevalence)

$P(\ominus | G) = 0.97$ (specificity)

$P(\oplus | B) = 0.98$ (sensitivity)

→ any 2 possible truth states (B, G)

Solution 1: 2x2 table ✓

METHOD 2:

$$\frac{P(B|\oplus)}{P(\text{not } B|\oplus)} = \left[\frac{P(B)}{P(\text{not } B)}\right] \cdot \left[\frac{P(\oplus|B)}{P(\oplus|\text{not } B)}\right] \circledast$$ (likelihood ratio)

$$\left(\begin{matrix}\text{posterior} \\ \text{odds in} \\ \text{favor of } B\end{matrix}\right) = \left(\begin{matrix}\text{prior} \\ \text{odds in} \\ \text{favor of } B\end{matrix}\right) \cdot \left(\begin{matrix}\text{Bayes factor} \\ \text{in favor of } B\end{matrix}\right)$$

$$= \left(\frac{0.01}{0.99}\right)\left(\frac{0.98}{0.03}\right)$$

$$\left(\begin{matrix}99 \text{ to } 1 \\ \text{prior odds} \\ \text{against } B\end{matrix}\right)\left(\begin{matrix}98 \text{ to } 3 \text{ odds} \\ \text{in favor of } B\end{matrix}\right)$$

$$= \frac{98}{(99)(3)} = \frac{98}{297}$$ (posterior odds in favor of B)

METHOD 3:

Bayes's theorem in probability form

$$P(B|\oplus) = \frac{P(B)\,P(\oplus|B)}{P(\oplus)}$$

annoying denominator

how to compute $P(\oplus)$?

(truth) $\{B, \text{not } B\}$ (unknown)

→ Unless (true) state of world may well be unobservable

(data) $\{\oplus, \ominus\}$

$P(B|\oplus)$ ← truth (unknown); ← data; ← search for truths on basis of data; ← statistical inference

data (observable)

(statistical) inference about data → prediction

$P(\oplus) = ?$ I don't know much about or all about $\oplus$ by itself; If I knew B or $\text{not } B$, bingo.

New creative laziness idea:

when $\mathcal{I}$ is hard, [get help] [extend the conversation]

find some other aspect of world $\mathcal{X}$ upon which $\mathcal{I}$ depends & predict $\mathcal{I}$ in 2 stages: $\{\mathcal{X}\}$ direct, $(\mathcal{I}|\mathcal{X})$ indirect

↓ data ($\mathcal{I}$) other info $\mathcal{X} = \{G, B\}$

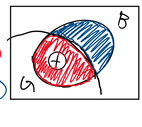
$P(\oplus) = P[\oplus \text{ and } (G \text{ or } B) \text{ or } \oplus \text{ and } B)]$

$= P(\oplus \text{ and } G) + P(\oplus \text{ and } B)$ $P(\oplus \text{ and } G)$

$= P(G)\,P(\oplus|G) + P(B)\,P(\oplus|B)$ $P(\oplus \text{ and } B)$

$= (0.99)(0.03) + (0.01)(0.98)$

$= (0.0297) + (0.0098) = \boxed{0.0395}$



⇒ So finally

$$P(B|\oplus) = \frac{P(B)\,P(\oplus|B)}{P(\oplus)} = \frac{(.01)(.98)}{.0395} = \frac{98}{395} = 0.25$$

DG Ch. 3 Random Variables

Given a sample space S for experiment $\mathcal{E}$, a (real-valued) random variable (rv) is a function from the mean-weird collection $\mathcal{C}$ of

Notation $P(\mathcal{I} = y) = P(\{s : \mathcal{I}(s) = y\})$

$P(\mathcal{I} = y)$ ← a possible value of $\mathcal{I}$ (outcome); ↑ random variable (process)