4/25/19

Thursday, April 25, 2019 1:32 PM

This time: PMF, PDF, CDF

new due date take-home test 1: May 5

next time: inverse CDF

f_Y keeps track of the probability associated with Y: $f_Y(y) = P(Y = y)$

<u>Powerball</u> $P(W_1 = w_1) = \begin{cases} \frac{1}{69} & w_1 = 1, 2, \dots, 69 \\ 0 & \text{otherwise} \end{cases}$

definition: for any two integers $a \le b$ a rv Y that's equally likely to be any of the values $\{a, a+1, \dots, b\}$ has the <u>uniform</u> <u>distribution</u> uniform $\{a, b\}$. Evidently

"is distributed as"

$Y \sim \text{uniform}\{a, b\} \longleftrightarrow Y$ chosen at <u>random</u> from $\{a, a+1, \dots, b\}$

$\sum_{y=a}^{b} f_Y(y) = 1$

pf <u>Binomial</u> <u>distribution</u>

$f_Y(y) = P(Y = y) = \begin{cases} \binom{n}{y} p^y (1-p)^{n-y} & \text{for } y = 0, 1, \dots, n \\ 0 & \text{else} \end{cases}$

parameters

$Y \sim \text{Binomial}(n, p)$

$X_i \overset{IID}{\sim} f_{X_i}(x_i)$

$\left(Y = \sum_{i=1}^{n} B_i \right) \sim \text{Binomial}(n, p).$

<u>CASE STUDY</u>: bank teller wait times

$N(t) = \#$ arrivals in $[0, t]$

or

inter-arrival times $T_1, T_2, \dots$

<u>Assumption 1</u>: the numbers of arrivals in any collection of disjoint time intervals are (mutually) independent.

<u>Assumption 2</u>: arrival process is smooth not lumpy

<u>Assumption 3</u>: nearly simultaneous arrivals are rare.

λ constant $\longleftrightarrow$ $N(t)$ is a stationary stochastic process

$f_Y(y \mid \lambda) = P(Y = y \mid \lambda) = \begin{cases} \frac{\lambda^y e^{-\lambda}}{y!} & \text{for } y = 0, 1, \dots \\ 0 & \text{else} \end{cases}$

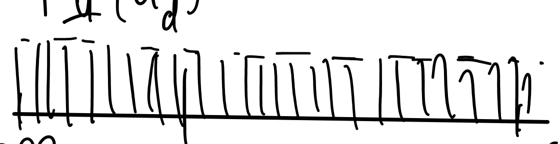
$\boxed{\text{Result}}$ Under assumptions 1-3 above, $N(t)$ is a Poisson process with rate parameter λ.

continuous uniform

$f_U(u) = \begin{cases} 1 & 0 \le u \le 1 \\ 0 & \text{else} \end{cases}$

$f_U(u_d)$

$\frac{1}{100}$



0.00 0.19 u_d 0.99 U_d

$P(0.0 \le U_d \le 0.19) = \sum_{u_d = 0.00}^{0.19} f_{U_d}(u_d) = 0.2$

<u>Definition</u>: A random variable (Y has a continuous distribution) if there exists a continuous non-negative function f_Y defined on $\mathbb{R}$ such that for every interval $[a, b]$ $P(a \le Y \le b) = \int_a^b f_Y(y)\,dy$

$f_Y(y) \ge 0$ & $\int_{-\infty}^{\infty} f_Y(y)\,dy = 1$

$$\boxed{P(Y = y) = 0 \quad \text{for all } -\infty < y < \infty}$$

$Y \sim \text{Uniform}(a, b)$

Density and probability are not the same thing

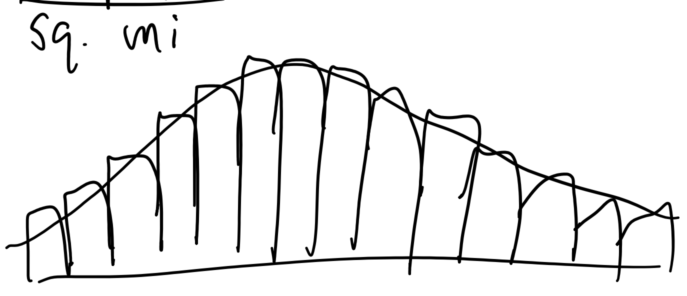
$P\left(a - \frac{\epsilon}{2} \le Y \le a + \frac{\epsilon}{2}\right) = \epsilon \cdot f_Y(a)$

$\boxed{f_Y(a)} = \boxed{\dfrac{P\left(a - \frac{\epsilon}{2} \le Y \le a + \frac{\epsilon}{2}\right)}{\epsilon}}$

density probability

ex. pop. density $= \frac{\#\text{ people}}{\text{sq. mi}}$

(probability concentration near a)



distribution