# 5/28/19

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Case 1: Discrete

Bernoulli

Def: If the $X_i$ in $X_1, X_2, \dots$ are IID Bernoulli $(p)$, then ...

Binomial

$f_X(x) = \binom{n}{x} p^x (1-p)^{n-x} \mathbb{1}_{\{0,1,\dots,n\}}$

$E(X) = n \cdot p \qquad V(X) = np(1-p)$

hypergeometric

| Sampling method | mean | variance |
|---|---|---|
| IID | $n\left(\frac{A}{A+B}\right)$ | $n\left(\frac{A}{A+B}\right)\left(\frac{B}{A+B}\right)$ |
| SRS | $n\left(\frac{A}{A+B}\right)$ | $n\left(\frac{A}{A+B}\right)\left(\frac{B}{A+B}\right)\left(\frac{T-n}{T-1}\right)$ |

$0 \le q = \frac{T-n}{T-1} \le 1$ finite population correction

($n$ fixed, $T\uparrow$) with a small sample from a large population, SRS = IID

Poisson

$E(X) = \lambda \qquad V(X) = \lambda$

$\frac{V(X)}{E(X)} = 1$ (variance-to-mean ration) (VTMR)

$\rightarrow$ when $\left(\begin{matrix} n \text{ is large} \\ p \text{ is close to } 0 \end{matrix}\right)$, Binomial$(n,p)$ = Poisson$(n \cdot p)$

Negative Binomial Distribution

$f_X(x \mid r, p) = \binom{r+x-1}{x} p^r (1-p) \dots$

MGF:

# CONTINUOUS:

Normal (Gaussian) Distribution:

$X \sim \text{Normal}(\mu, \sigma^2)$

PDF

$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right]$

properties:

$X \sim \text{Normal}(\mu, \sigma^2) \qquad E(X) = \mu$

MGF: $\psi_X(t) = \exp\left(\mu t + \frac{\sigma^2 t^2}{2}\right)$

center of symmetry, mean, median, mode $\overset{\text{all}}{=} \mu$

Consequences:

① $X \sim \text{Normal}(\mu, \sigma^2) \quad Y = aX + b \quad \left(\begin{matrix} a \neq 0 \\ b \end{matrix}\right)$ fixed constants

$Y \sim \text{Normal}(a\mu + b, a^2\sigma^2)$

Normality is preserved under linear transformation

The normal dist. with $\mu = 0$ and $\sigma = 1$ is called the standard normal dist.

The PDF of $X \sim \text{Normal}(0,1)$ is

$\phi_X(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right)$

CDF is $\Phi(x) \triangleq \int_{-\infty}^{x} \phi_X(t)\, dt$

Empirical Rule

Part 1: $(\mu \pm \sigma) = 68\%$

Part 2: $(\mu \pm 2\sigma) = 95\%$

Part 3: $(\mu \pm 3\sigma) = 99.7\%$

for data: $z = \frac{y - \bar{y}}{s} = SU$

for rv's: $Z_i = \frac{Y - \mu}{\sigma} = SU$