4/4/19

Thursday, April 4, 2019 1:26 PM

this time: Case study; experiments ⇒ read DS Ch. 1

next time: events; sample spaces; set theory ⇒ Office hours set this quarter Starting on Fri 5 Apr. 19 (see course web page)

Subject email: "AMS_131", "AMS131"

Case Study 1: Tay-Sachs Disease

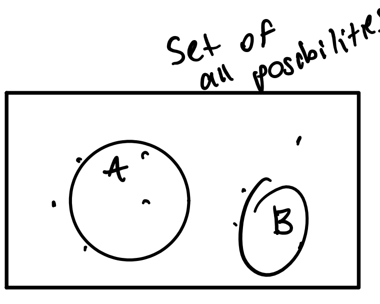
P(1 or more T-S in 5 children, both parents carriers) = ?

P(A) → true/false prop. → set

↑ probability of

Set of all possibilities

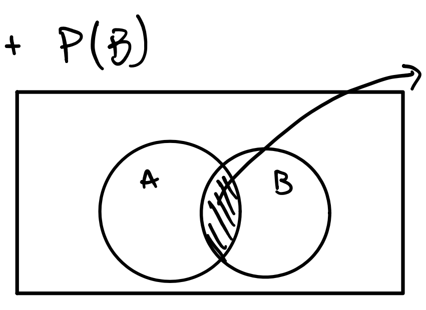
John Venn Diagram



⇒ no overlap ↳ P(A and B) = 0

P(▭) = 1 = 100% $P(A) = \frac{\text{Ⓐ}}{\square\ (1)}$

P(A or B) = P(A) + P(B)



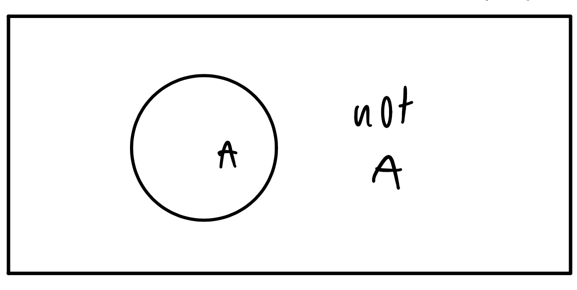
intersection (overlap) (A and B)

$\underbrace{P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)}_{\text{addition rule for } \boxed{OR}}$

$0 \le P(A) \le 1$
↑ impossibility ↑ certainty

A, B no overlap: A, B mutually exclusive

Ω



P(A or (not A)) = 1
= P(A) + P(not A)

P(A) = 1 − P(not A)
direct indirect

$P(A \text{ and } B) \overset{?}{=} P(A) \ ? \ P(B)$

2 cases

population $n=3 \begin{bmatrix} 1 \\ 2 \\ 9 \end{bmatrix} \xrightarrow{\text{at random}}$ sample $\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix}$ $n=2$

$P(Y_1 = 9 \text{ and } Y_2 = 9) = ?$

IID, SRS

IID (at random with replacement)

	1	2	9
1	(1,1)	(1,2)	(1,9)
2	(2,1)	(2,2)	(2,9)
9	(9,1)	(9,2)	(9,9)

ELM? → yes

9 elemental outcomes (EO's)

$P(Y_1 = 9 \text{ and } Y_2 = 9) = \frac{1}{9}$

$P(Y_1 = 9) = \frac{1}{3} = \frac{3}{9}$ $P(Y_2 = 9) = \frac{1}{3} = \frac{3}{9}$

$P(Y_1 = 9 \text{ and } Y_2 = 9)$
$= \frac{1}{9} = P(Y_1 = 9) \cdot P(Y_2 = 9)$

SRS (at random without replacement)

	1	2	9
1	(1,1)	(1,2)	(1,9)
2	(2,1)	(2,2)	(2,9)
9	(9,1)	(9,2)	(9,9)

$P(Y_1 = 9 \text{ and } Y_2 = 9) = 0$

$P(Y_1 = 9) = \frac{1}{3} = \frac{2}{6}$ $P(Y_2 = 9) = \frac{1}{3} = \frac{2}{6}$

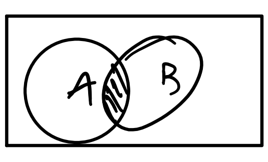
$P(Y_1 = 9 \text{ and } Y_2 = 9) = 0 \ne \underbrace{\frac{1}{3}}_{P(Y_1=9)} \cdot \underbrace{\frac{1}{3}}_{P(Y_2=9)}$

Abraham de Moivre (1705)

Thomas Bayes (1760)

Conditional Probability

$P(A) = \frac{\text{Ⓐ}}{\square\ (1)}$



$P(A \mid B) = \frac{\text{A and B}}{\text{B}}$
↑ "given"

$P(A \mid B) = \begin{cases} \frac{P(A \text{ and } B)}{P(B)} & \text{if } P(B) > 0 \end{cases}$, P(B) = 0 undefined

general product rule for AND

$P(A \text{ and } B) = P(B) \cdot P(A \mid B)$
$= P(A) \cdot P(B \mid A)$

chain rule for AND

$P(B \mid A) = \frac{P(B \text{ and } A)}{P(A)} = \frac{P(A \text{ and } B)}{P(A)}$

Bayesian

⇒ $P(Y_1 = 9 \text{ and } Y_2 = 9) =$
$P(Y_1 = 9) \cdot P(Y_2 = 9 \mid Y_1 = 9)$
$= \frac{1}{3} \cdot 0 = 0$ ✓

$P_{IID}(Y_1 = 9 \text{ and } Y_2 = 9) =$
$P_{IID}(Y_1 = 9) \cdot P_{IID}(Y_2 = 9 \mid Y_1 = 9)$
$= P_{IID}(Y_1 = 9) \cdot P_{IID}(Y_2 = 9)$

If info about A doesn't change chances about B, & vice versa,
$\boxed{\text{def.}}$ A, B are independent

Frequentist Definition

A, B are independent iff (if and only if) P(A and B) = P(A) · P(B)

I I D
independent, identically distributed

T-S P(1 or more T-S babies)

# of T-S babies

1
2
3 } 6 "EOs"
4
5

if ELM applies P( ) = $\frac{5}{6}$ ✗

but ELM doesn't apply

P(1) > P(0) bogus

P(1 or more T-S) = P(exactly 1 TS or … or exactly 5)

P(exactly 1) + … + P(exactly) = 0

↳ P(1 or more T-S) = 1 − P(0 T-S)

= 1 − P(not T-S on 1st $\boxed{\text{and}}$ not T-S on 2nd $\boxed{\text{and}}$ … $\boxed{\text{and}}$ not T-S on 5th)

IID
$= 1 - P(\text{not T-S on 1st}) \cdot P(\text{not T-S on 2nd}) \cdots P(\text{not T-S on 5th}) = 1 - \left(1 - \frac{1}{4}\right)\left(1 - \frac{1}{4}\right) \cdots \left(1 - \frac{1}{4}\right)$

$P(1 \text{ or more T-S}) = 1 - \left(1 - \frac{1}{4}\right)^5 = 76\%$

→ we just made a calculation with the Binomial distribution