This time: Large random Sample
Next time: Markov Chains

**IID random sample** $X_1, X_2, \ldots, X_n$

$M = E(X)$

**Sample Mean**: $\bar{X} = \frac{1}{n} \sum_{i=1}^{n} X_i$

Would we use the Law of Large Numbers to improve the range $M$ as $n$ increases? How to quantify that idea?

**Markov Inequality**

$$P(X \geq c) \leq \frac{E(X)}{c}$$

$E(X) = \mu$, so

$$P(X \geq \mu) \leq \frac{\mu}{c}$$

**Chebyshev Inequality**

$$P(|X - E(X)| \geq \epsilon) \leq \frac{V(X)}{\epsilon^2}$$

$$\Rightarrow \quad \text{BACK TO}$$

$$E(X) = M \quad V(X) = \sigma^2 \quad \epsilon = \frac{x_0 - M}{\sigma}$$

$$E(\frac{1}{n} \sum_{i=1}^{n} X_i - M)^2 \geq \frac{\epsilon^2}{\sigma^2}$$

**rearranged**

$$P\left(\frac{1}{n} \sum_{i=1}^{n} X_i - M \geq \frac{\epsilon}{\sigma}\right) \geq 1 - \frac{\epsilon^2}{\sigma^2}$$

**Example:**

A sequence $X_1, X_2, \ldots$, $X_n, \ldots$ is said to converge in probability to a constant $a$ if for all $\epsilon > 0$, $\lim_{n \to \infty} P(|X_n - a| < \epsilon) = 1$.

This is denoted $X_n \to a$.

**Law of Large Numbers**

$X \sim \text{iid}$ a dist with mean $M$

**Central Limit Theorem**

$X_1, X_2, \ldots$ any dist with mean $M$ and finite variance $\sigma^2 \leq \sigma^2$, $n \to \infty$

$$\frac{\sum_{i=1}^{n} X_i - Mn}{\sigma / \sqrt{n}} \to N(0, 1)$$

**Central Limit Theorem**

Def: $X_1, X_2, \ldots$ a sequence of $X$, let $S_n = \sum_{i=1}^{n} X_i$

- If there exists a constant $\tau$ such that $\lim_{n \to \infty} F_n(x) = \tau(x)$ for all $x$ at which $\tau(x)$ is defined, then property $\tau$ has

- $E(S_n)$ and $\text{Var}(S_n)$:

$$E(S_n) = nM$$

$$\text{Var}(S_n) = \sum_{i=1}^{n} \text{Var}(X) = n\sigma^2$$

**Example**

$$E(g(S_n)) = g(nM)$$

$$\text{Var}(g(S_n)) = \left[ g'(\mu_S) \right]^2 \text{Var}$$

**Delta Method**

$$E(g(X)) = g(\mu_X)$$

$$\text{Var}(g(X)) = (g'(\mu_X))^2 \text{Var}$$

**Continuity Conditions**