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AMS 7: Discussion Section 4

1. A big problem facing banks and merchants alike is determining potential customers' credit-worthiness. You've probably had the experience of trying to use a credit card to pay for something — store owners typically have a little electronic machine they pass your card through, and information stored in the magnetic strip on the back (mainly your credit card account number) is transmitted over a telephone line to a central *credit-verification system* somewhere. This system uses a computer program to decide if your purchase should be approved, based on factors like how often it thinks your card has been used lately and how recently it thinks you've paid your bill. If you pass this invisible screening, a little green light goes on back in the store, and you walk out with your purchase.

Of course, any computer-based system of this kind makes mistakes sometime, because of faulty information or bad programming: sometimes credit cards that are good are declared bad, sometimes vice versa. Events of this type look random to the people at the bank trying to figure out why they happen — at least until the causes of the mistakes are determined — so it makes sense to talk about the *probability* that a bad credit card is declared good, and the probability that a good card is judged bad. Standard terminology is to call the kind of mistake in which you declare a good credit card bad a *false positive*, and the other kind of mistake — in which you label a bad credit card good — a *false negative*. (Evidently the people who made up this terminology were thinking of "positive" in this context as equivalent to "calling a credit card bad" and "negative" as amounting to "calling the card good," which is a little perverse, but there it is.)

People evaluate the quality of credit-screening systems of this type by running tests in which (a) they attempt a number of fake "purchases" with some credit cards that are known to be good and some others that are known to be bad, and (b) they look to see how often the system gets it right. Suppose that in one test of this type on the system we're going to look at, 97% of the test "purchases" with cards known to be good were labeled good by the system, and 98% of the "purchases" in which the test cards were bad were declared bad by the system. Suppose further that the system is to be used in a market in which about 1% of all attempted purchases are with bad credit cards. Somebody now walks into a store using this credit verification system and tries to make a purchase by credit card, and the system comes back with a negative opinion about this person's credit-worthiness. Show that the conditional probability the card is indeed bad, given the system's "diagnosis," is only about 25%! How can you make sense of this result, assuming that the people who designed the screening system aren't stupid? Explain briefly.

2. As you may know, there are a number of different subway lines in London, some of which run in parallel under the same streets. The builders of the Underground arranged this by stacking the subway tunnels for the lines under each other, sometimes two or three deep. At many Underground stations, to get to the deepest tunnels you take a long escalator ride. For instance, at the Pimlico station the escalator down to the deepest tunnel is like a