Discussion section 5 | Practice with PMFs, PDFs, CDFs and inverse CDFs | AMS 131, 1 May 19

Discrete (PMFs) | Continuous (PDFs)

Bernoulli ← (discrete) Uniform

Binomial CDF ✓

Poisson

Hypergeometric

(Continuous) Uniform

triangular CDF ✓

Exponential

⋮

~~CDF of Y~~

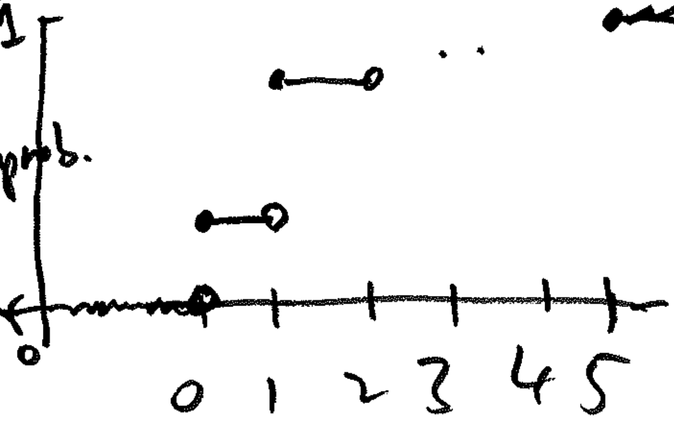
for any (discrete) rv Y, $F_Y(y) = P(Y \le y)$

Binomial (n, p) $\quad Y \sim \text{Binomial}(n, p)$

(1) PMF $f_Y(y) = \begin{cases} \binom{n}{y} p^y (1-p)^{n-y} & \text{for } y = 0, 1, \dots, n \\ 0 & \text{else} \end{cases}$

$n = 5$
$(p = \frac{1}{4})$ (Tay rocks)

$y \to$



CDF of Binomial$(5, \frac{1}{4})$

$$F_Y(y) = P(Y \leq y)$$

Exponential(λ)

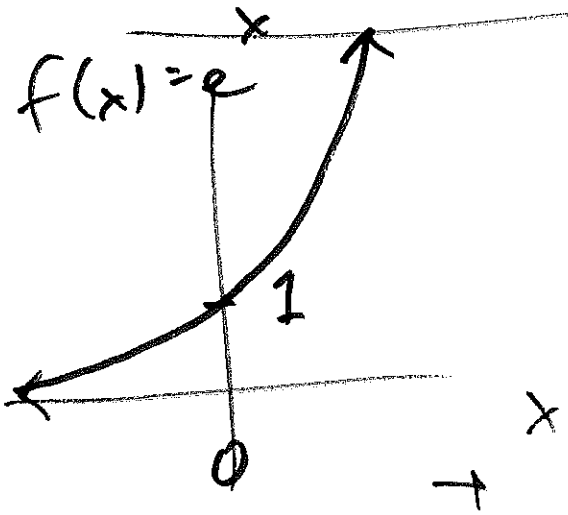
$(\lambda > 0)$
(rate)

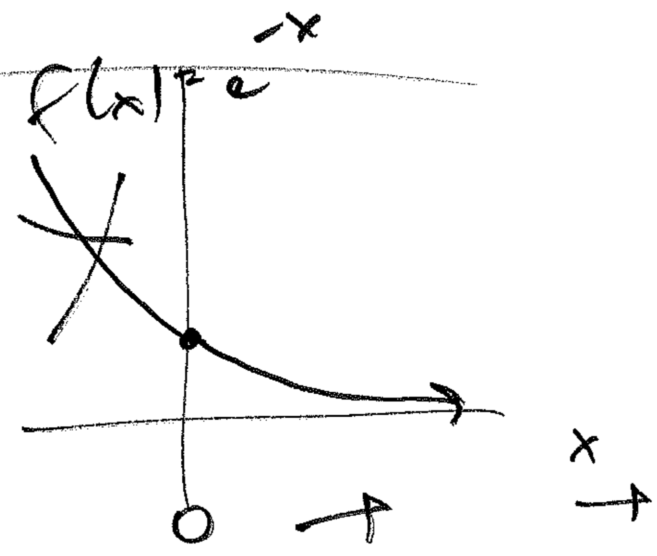
$\frac{1}{\beta} e^{-\frac{y}{\beta}}$ $\quad \beta = \frac{1}{\lambda} \leftrightarrow \lambda = \frac{1}{\beta}$

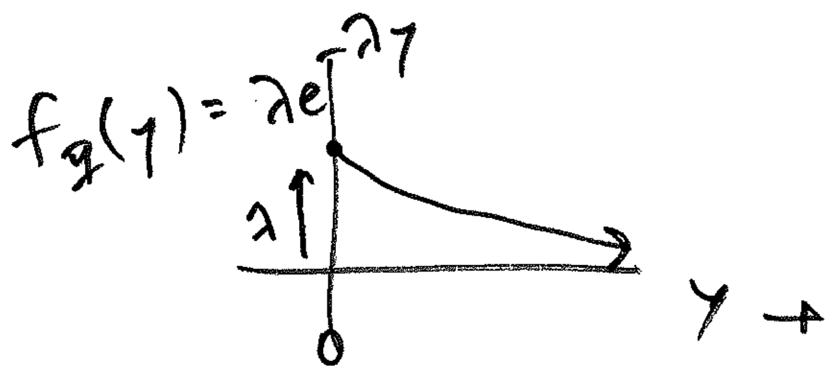
$Y \sim$ Exponential(λ) $\leftrightarrow$

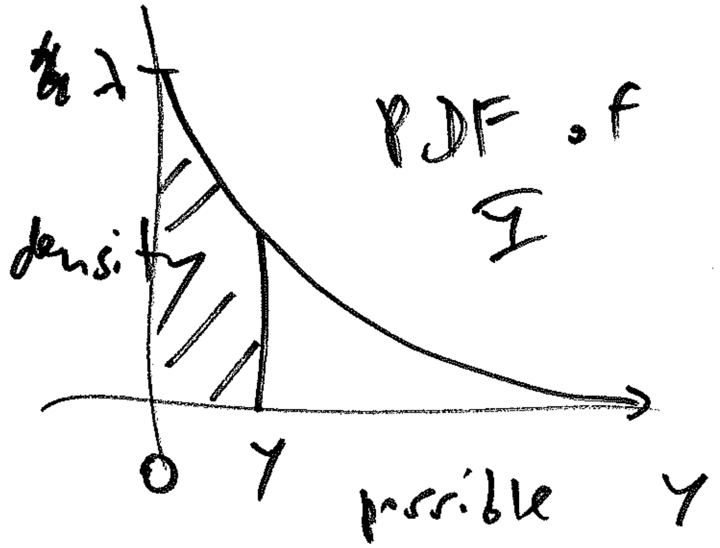
$$f_Y(y) = \begin{cases} \lambda e^{-\lambda y} & \text{for } y > 0 \\ 0 & \text{else} \end{cases}$$

PDF



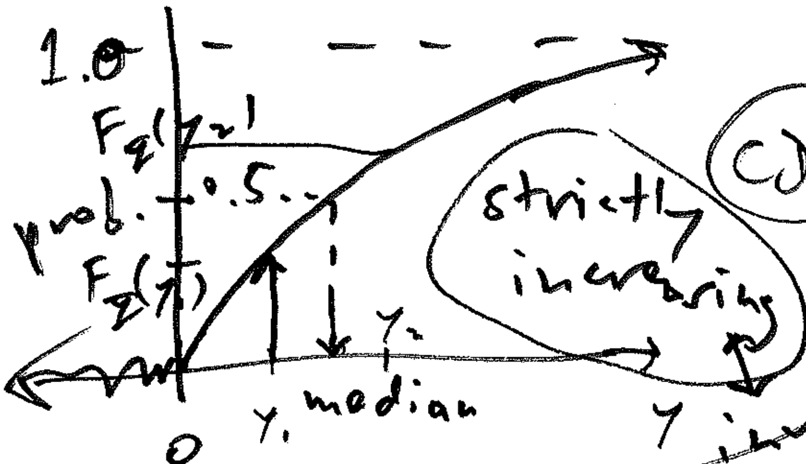






PDF of Y

$$F_Y(y) = \begin{cases} 0 & \text{for } y \le 0 \\ 1 - e^{-\lambda y} & y \ge 0 \end{cases}$$



CDF

for $y > 0$

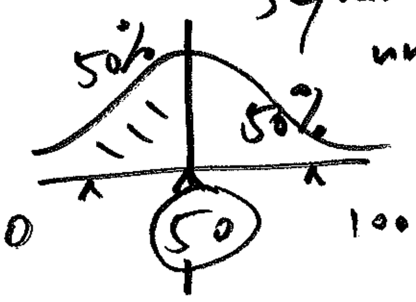
$$F_Y(y) = P(Y \le y)$$

$$= \int_0^y f_Y(t)\,dt$$

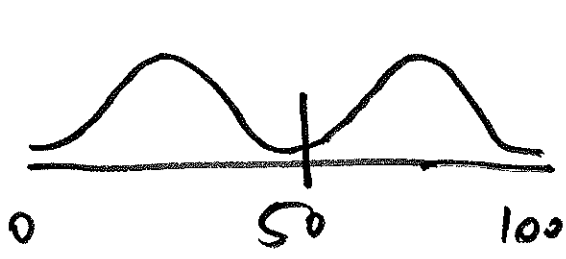
$$= \int_0^y \lambda e^{-\lambda t}\,dt$$

$$= 1 - e^{-\lambda y}$$

symmetric unimodal



point of symmetry = mean = median = mode



median : inverse CDF

for $y > 0$ $\quad F_Y(y) = 1 - e^{-\lambda y} = p$

solve for y: $\quad 1 - p = e^{-\lambda y}$

$$\log(1-p) = -\lambda y$$

$$F_Y^{-1}(p) = y = -\frac{1}{\lambda}\log(1-p)$$

inverse CDF