Discussion Section 7

AMS 131
15 May 19

①

X discrete rv
PMF $f_X(x)$
support S

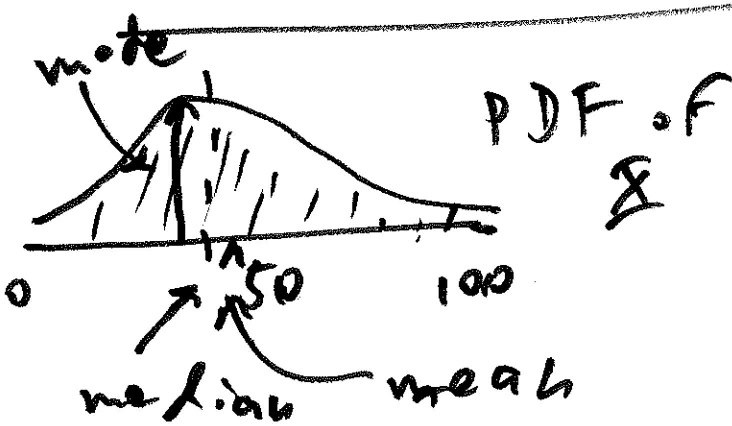
$$E(X) = \sum_{\text{all } x \in S} x \, f_X(x)$$

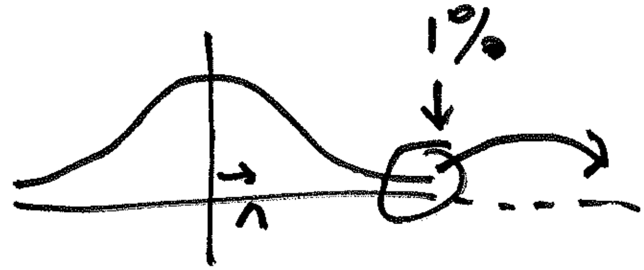
expected value of X

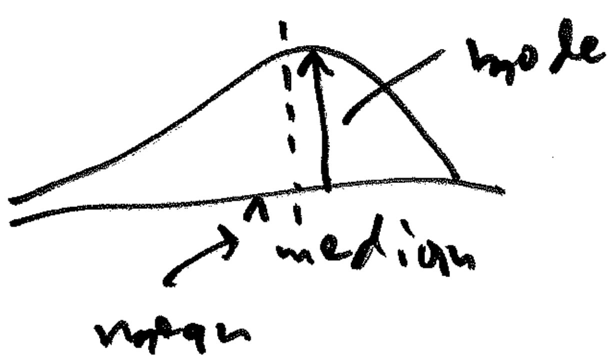
X continuous rv, PDF $f_X(x)$
support S

$$E(X) = \int_S x \, f_X(x) \, dx$$

mean of X







discrete $(X \mid p) \sim \text{Bernoulli}(p)$

$$\leftrightarrow \text{PMF} \quad f_X(x) = \begin{cases} \text{prob.} & \text{support} \\ p & x = 1 \\ 1-p & 0 \\ 0 & \text{else} \end{cases}$$

$S = \{0, 1\}$

$$E(X) = \sum_{\substack{\text{all } x \\ \text{in } S}} x \, f_X(x) = 0 \cdot (1-p) + 1 \cdot p = p$$

$(X \mid n, p) \sim \text{Binomial}(n, p)$

$$\leftrightarrow \text{PMF} \quad f_X(x) = \begin{cases} \text{prob} & \text{support} \\ \binom{n}{x} p^x (1-p)^{n-x} & x = 0, 1, \dots, n \\ 0 & \text{else} \end{cases}$$

$$E(X) = \sum_{\substack{\text{all } x \\ \text{in } S}} x \, f_X(x) = \sum_{x=0}^{n} x \binom{n}{x} p^x (1-p)^{n-x} \overset{\boxed{\text{Wd}}}{=} np$$

$$E(X^2) = \sum_{x=0}^{n} x^2 \binom{n}{x} p^x (1-p)^{n-x} = p[(n-1)np + n]$$

$(\underline{X} \mid \lambda) \sim \text{Poisson}(\lambda) \quad \longleftrightarrow \quad \mathcal{S} = \{0, 1, \ldots\}$

$$\text{PMF} \quad f_{\underline{X}}(x) = \begin{cases} \dfrac{\lambda^x e^{-\lambda}}{x!} & \text{for } x = 0, 1, \ldots \\ 0 & \text{else} \end{cases}$$

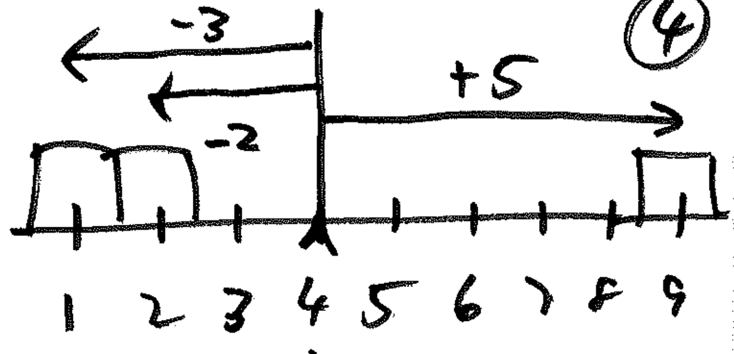
$$E(\underline{X}) = \sum_{x=0}^{\infty} x \, \frac{\lambda^x e^{-\lambda}}{x!} \overset{\boxed{\text{wd}}}{=} \lambda$$

$$E(\underline{X}^2) = \sum_{x=0}^{\infty} x^2 \, \frac{\lambda^x e^{-\lambda}}{x!} \overset{\boxed{\text{wd}}}{=} \lambda(\lambda + 1)$$

→ sum x lambda^x e^(-lambda)/x!;
x = 0 to infinity

measures of spread

$$\begin{bmatrix} 1 \\ 2 \\ 9 \end{bmatrix} \begin{matrix} y_1 \\ \vdots \\ y_n \end{matrix}$$



mean 4

$$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} n=3$$

spread 0

$$\begin{bmatrix} c \\ c \\ \vdots \\ c \end{bmatrix} \uparrow n \downarrow$$

spread 0

$$\begin{pmatrix} 1 \\ 2 \\ 9 \end{pmatrix} \xrightarrow{\text{subtract } 4} \begin{bmatrix} -3 \\ -2 \\ +5 \end{bmatrix}$$

$$\begin{pmatrix} y_1 \\ | \\ y_n \end{pmatrix} \xrightarrow{\text{subtract } \bar{y}} \begin{bmatrix} y_1 - \bar{y} \\ y_2 - \bar{y} \\ \vdots \\ y_n - \bar{y} \end{bmatrix}$$

deviations

$$\begin{bmatrix} |y_1 - \bar{y}| \\ \vdots \\ |y_n - \bar{y}| \end{bmatrix}$$

$$\begin{bmatrix} |-3| \\ |-2| \\ |+5| \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 5 \end{bmatrix}$$

mean $\frac{10}{3} = 3.3$

MAD = mean absolute deviation (from the mean)

$$\text{MAD} = \frac{1}{n} \sum_{i=1}^{n} |y_i - \bar{y}|$$

1785 (Laplace)

$$\begin{bmatrix} \$1 \\ \$2 \\ \$9 \end{bmatrix} \xrightarrow{\text{subtract } \$4} \begin{bmatrix} -\$3 \\ -\$2 \\ +\$5 \end{bmatrix} \xrightarrow{\text{square}} \begin{bmatrix} (-3)^2 = +9 \\ (-2)^2 = +4 \\ (+5)^2 = +25 \end{bmatrix}$$

(Gauss 1800)

$$\begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} \xrightarrow{\text{subtract } \bar{y}} \begin{bmatrix} y_1 - \bar{y} \\ \vdots \\ y_n - \bar{y} \end{bmatrix} \xrightarrow{\text{square}} \begin{bmatrix} (y_1 - \bar{y})^2 \\ \vdots \\ (y_n - \bar{y})^2 \end{bmatrix}$$

$(\$)^2$

$$\frac{1}{n} \sum_{i=1}^{n} (y_i - \bar{y})^2 = \underline{\text{variance}} \text{ of } \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix}$$

$$\sqrt{\frac{1}{n} \sum_{i=1}^{n} (y_i - \bar{y})^2} = \sqrt{\text{variance}}$$

$$= \text{standard deviation (SD)}$$

K. Pearson (1900)

prob. (PMF)

$$\sum_{i=1}^{n} \left(\frac{1}{n}\right) \underline{(y_i - \bar{y})^2}$$

discrete Y

$$\text{r.v. } Y \to \underset{(\text{Var})}{V(Y)} = \sum_{\text{all } y \text{ in } S} (y - \bar{y})^2 f_Y(y)$$

$$\bar{Y} = \frac{1}{n}\sum_{i=1}^{n} Y_i \leftrightarrow E(Y) = \sum_{\substack{\text{all } y \text{ in} \\ S}} y\, f_Y(y)$$

$$V(Y) = E\left[(Y - \bar{Y})^2\right]$$

$$= E\left[Y - E(Y)\right]^2$$

$$SD(Y) = \sqrt{V(Y)}$$