AMS 131
16 May 19

①

this time: $\underline{\underline{E}}(X)$, $\underline{\underline{V}}(X)$, $\underline{\underline{C}}(X, Y)$, $\underline{\underline{SD}}(X)$,

bedrock in data science

next time: $\underline{\underline{\rho}}(X, Y)$

Support S_X

X discrete, PMF $f_X(x)$, →

$\underline{\underline{E}}(X) = \mu = \sum_{\substack{\text{all } x \\ \text{in } S_X}} x \, P(X = x)$

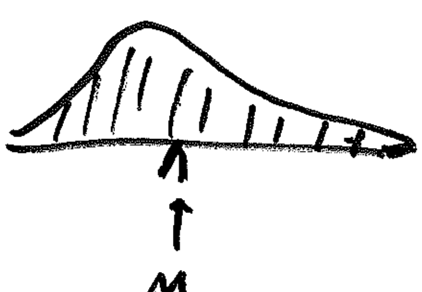
mean

"mew"

$= \sum_{\substack{\text{all } x \\ \text{in } S_X}} x \, f_X(x)$

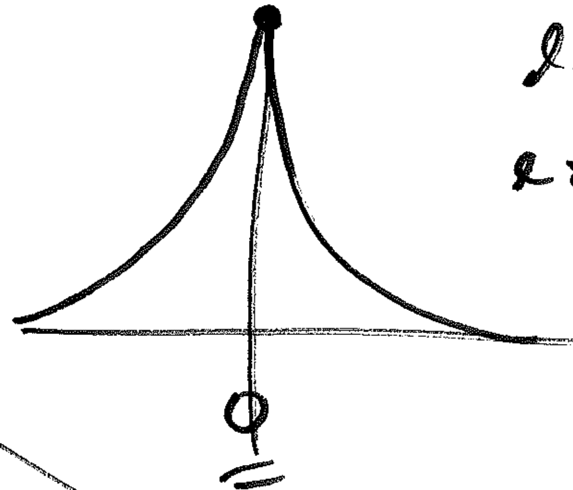
X continuous, PDF $f_X(x)$,

support S_X → $E(X) = \mu = \int_{S_X} x \, f_X(x) \, dx$



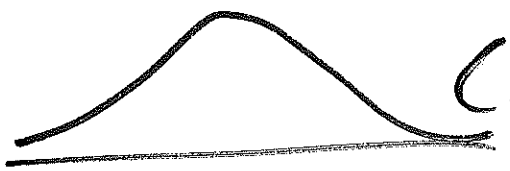
✓ center	spread	shape
mean median mode		

$E|X-\mu|$



double exponential (Laplace) PDF

$E(X-\mu)^2$

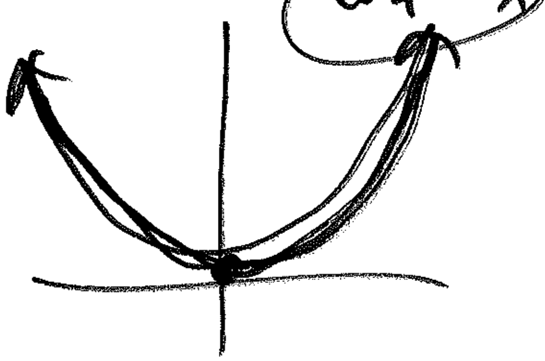


Gaussian (normal) PDF ← de Moivre

$$V(X) = E(X^2) - \left[E(X)\right]^2$$

$g(x) = x^2$

convex



by Jensen

$$\underbrace{E\left[g(X)\right]}_{\parallel} \ge g\left[E(X)\right]$$

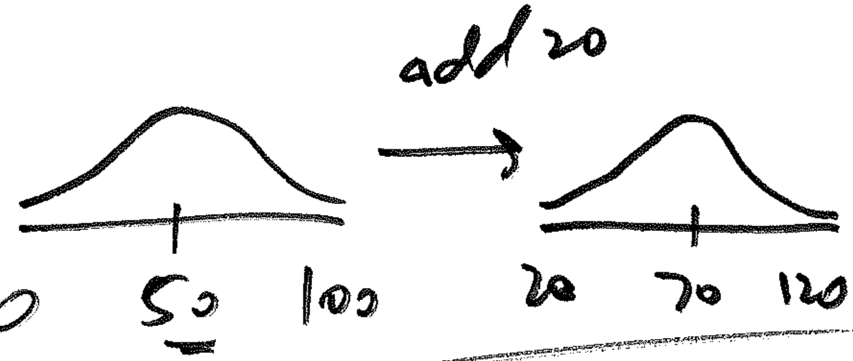
$$E(X^2) \ge \left(E(X)\right)^2$$

$$\therefore \underbrace{E(X^2) - \left[E(X)\right]^2}_{= V(X)} \ge 0$$

$$V(Y) = V(aX+b) = a^2V(X)$$

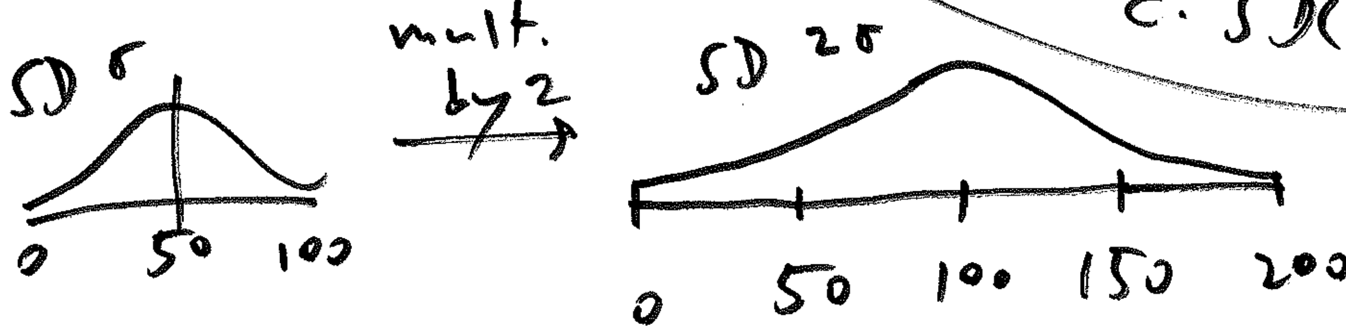
$$Y = aX + b$$

constants



$$\boxed{V(X+c) = V(X)}$$

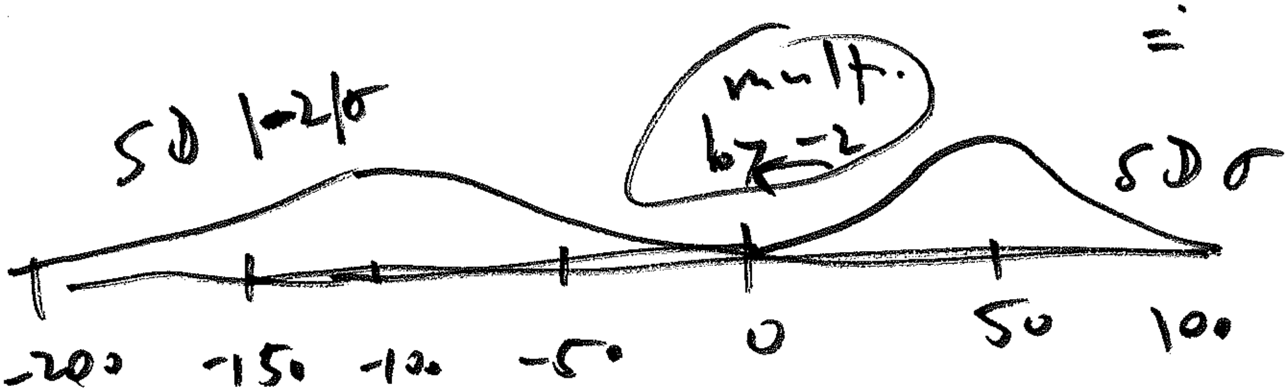
$$SD(cX) \overset{?}{=} c \cdot SD(X)$$



$$E(cX) = cE(X)$$

$$V(cX) = c^2V(X)$$

$$SD(-2X) \overset{?}{=} SD(X)$$

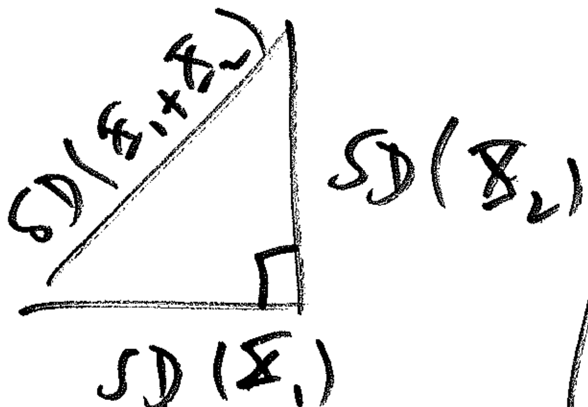


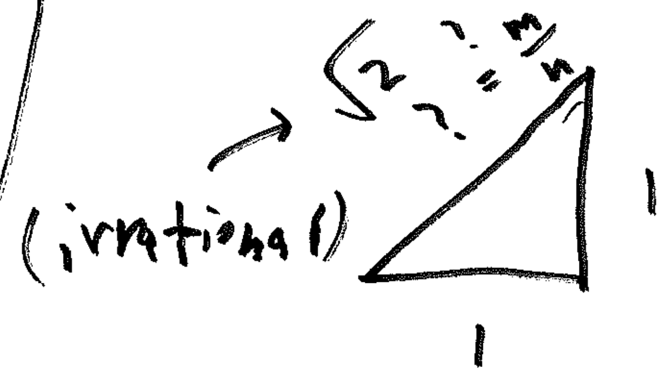
X_1, X_2 independent $\rightarrow$

$$V(X_1 + X_2) = V(X_1) + V(X_2)$$

$$\sqrt{V(X_1 + X_2)} = \sqrt{V(X_1) + V(X_2)}$$

$$SD(X_1 + X_2) = \sqrt{[SD(X_1)]^2 + [SD(X_2)]^2}$$

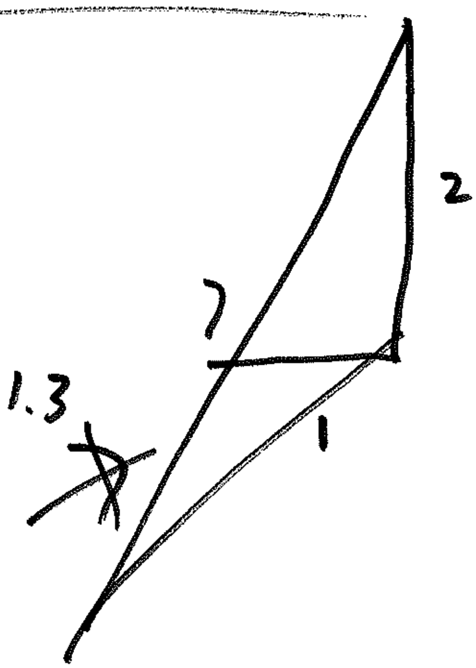




$SD(X_1) = 1$

$SD(X_2) = 2$

$SD(X_1 + X_2) = 1.3$

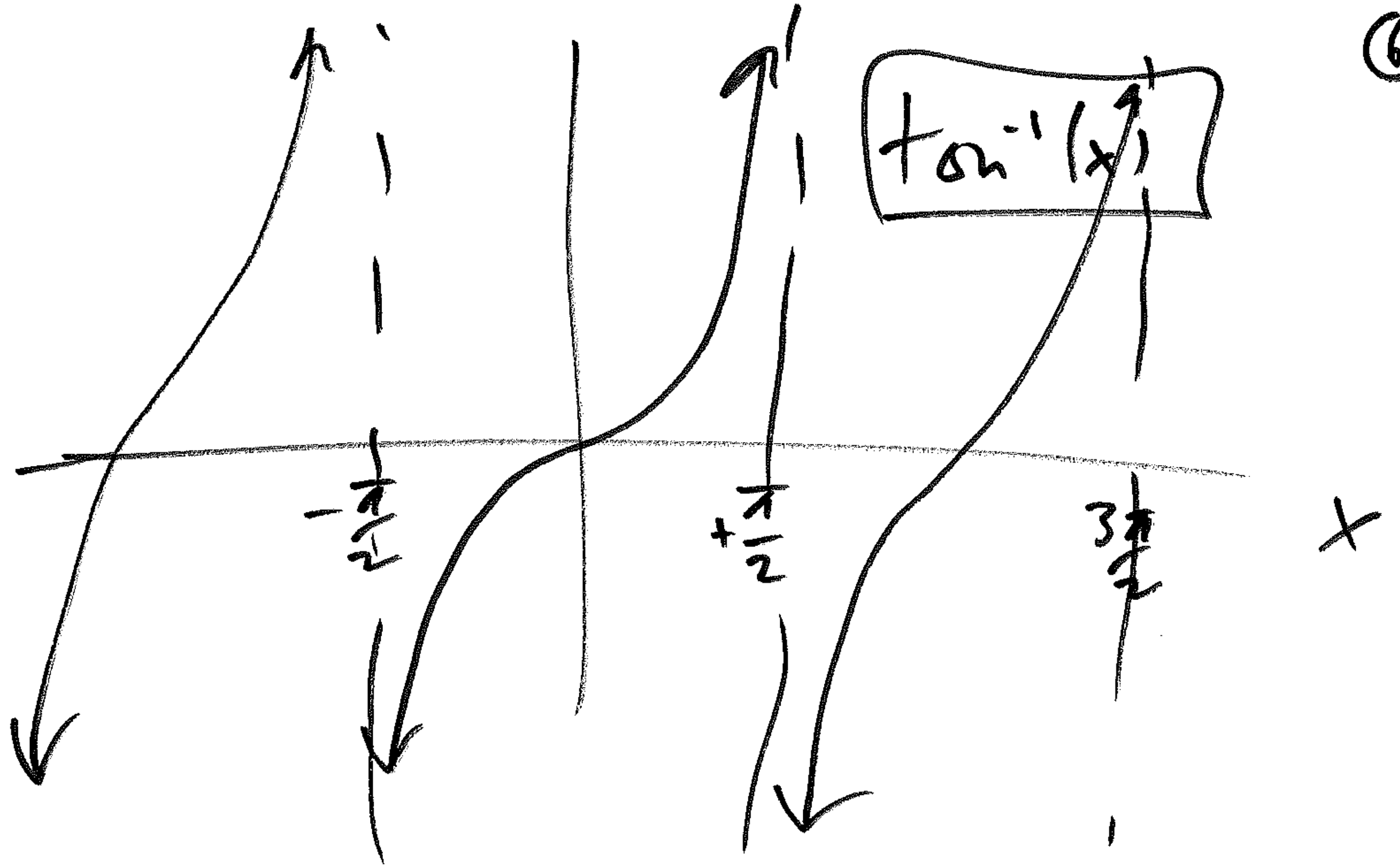


X_1, X_2 indep. $\to$ $\boxed{V(X_1 - X_2)} =$ 5

$= V(X_1) + V(-X_2)$

$= V(X_1) + (-1)^2 V(X_2)$

$V(X_1 + X_2) = V(X_1) + V(X_2)$



tan⁻¹(x)
−π/2
+π/2
3π/2
x