Discussion section week 8

DS p. 255 # 12

AMS 131
22 May 19

①

$$f_{XY}(x,y) = \begin{cases} \frac{x+y}{3} & \text{for } 0 \le x \le 1,\ 0 \le y \le 2 \\ 0 & \text{else} \end{cases}$$

(1)(F)

$V(2X - 3Y + 8) =$

$V(2X - 3Y) =$

$V(2X) + V(-3Y)$

$+ 2C(2X, -3Y)$

$= 2^2 V(X) + (-3)^2 V(Y)$

$+ 2 \cdot 2 \cdot (-3) C(X, Y)$

$= 4V(X) + 9V(Y) - 12C(X,Y)$

$V(W + c) = V(W)$

$V(cW) = c^2 V(W)$

$V(U + W) = V(U) + V(W) + 2C(U, W)$

$V(W) = E(W^2) - (E(W))^2$

$C(cU, W) = E(cUW) - E(cU)E(W)$

$= cE(UW) - cE(U)E(W)$

$= c[E(UW) - E(U)E(W)] = c \cdot C(U, W)$

to get $V(X) = E(X^2) - [E(X)]^2$, need ②

to extract marginal for X:

$$f_X(x) = \begin{cases} ⊛ & \text{for } 0 \le x \le 1 \\ 0 & \text{else} \end{cases}$$

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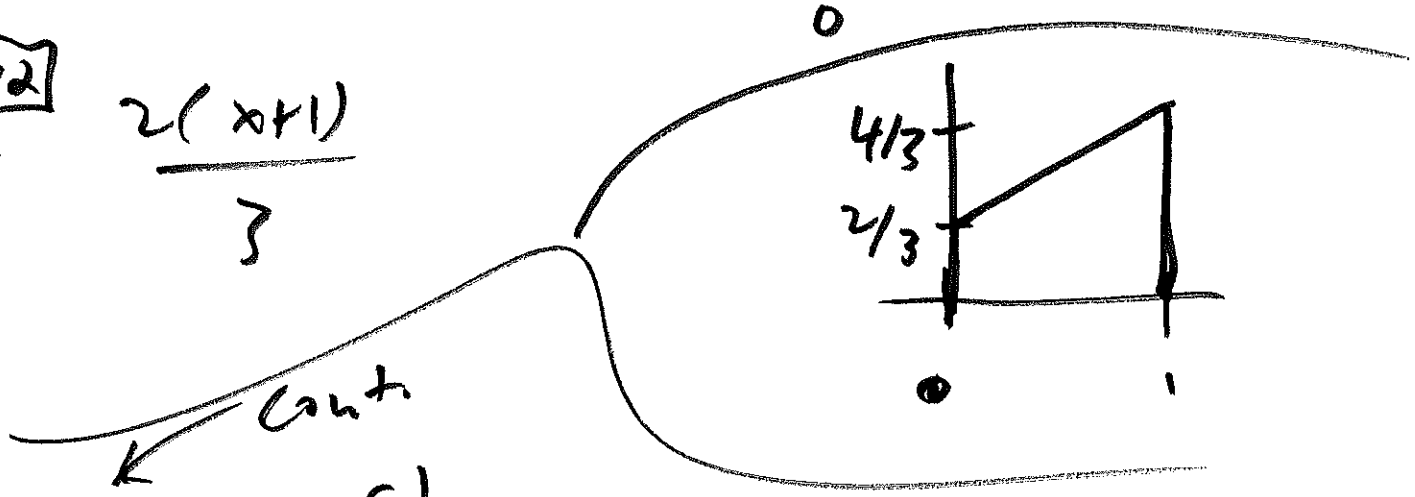
for $0 \le x \le 1$ $f_X(x) = \int_0^2 \frac{x+y}{3}\, dy = ⊛$

W2

$= \frac{2(x+1)}{3}$



$$E(X) = \int_0^1 x\, f_X(x)\, dx = \int_0^1 x \frac{2(x+1)}{3}\, dx = \frac{5}{9}$$

$$E(X^2) = \int_0^1 x^2 f_X(x)\, dx = \int_0^1 x^2 \frac{2(x+1)}{3}\, dx = \frac{7}{18}$$

$$V(X) = E(X^2) - \left(E(X)\right)^2$$

$$= \frac{7}{18} - \left(\frac{5}{9}\right)^2$$

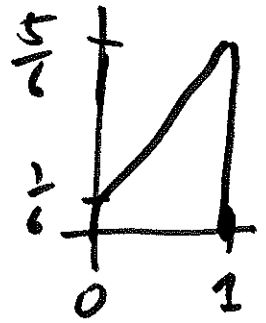
$$= \frac{13}{162}$$

$V(Y) = E(Y^2) - \left(E(Y)\right)^2$

need marginal for Y

$$f_Y(y) = \begin{cases} (**) & \text{for } 0 \le y \le 2 \\ 0 & \text{else} \end{cases}$$

$$(**) = \int_0^1 \frac{x+y}{3}\,dx = \frac{2y+1}{6}$$



$$E(Y) = \int_0^2 y\left(\frac{2y+1}{6}\right)dy = \frac{11}{9}$$

$$E(Y^2) = \int_0^2 y^2\left(\frac{2y+1}{6}\right)dy = \frac{16}{9}$$

$$V(Y) = E(Y^2) - (E(Y))^2$$

$$= \frac{16}{9} - \left(\frac{11}{9}\right)^2 = \frac{23}{81}$$

$$C(X, Y) = \boxed{E(XY)} - E(X)\,E(Y)$$

$$E(XY) = \int_0^1 \int_0^2 xy\, f_{XY}(x, y)\, dy\, dx$$

$$\left(\frac{x+y}{3}\right) = \frac{2}{3}$$

$$C(X, Y) = \frac{2}{3} - \left(\frac{5}{9}\right)\left(\frac{11}{9}\right) = -\frac{1}{81}$$

$$V(2X - 3Y + 8) =$$

$$4 \cdot \frac{13}{162} + 9 \cdot \frac{23}{81} - 12\left(-\frac{1}{81}\right) = \frac{245}{81}$$

$$\doteq 3.02$$

DS p. 255 #3

$X \sim \text{Uniform}(-2, +2)$

$Y \triangleq X^6$

$E(X) = 0$

show $\rho(X, Y) = 0$ ✓

$$\rho(X, Y) = \frac{C(X, Y)}{SD(X) \cdot SD(Y)}$$

$$\| \quad 0$$

$$C(X, Y) = E(XY) - E(X)E(Y)$$

$$\| \quad 0 = E(X^7) - E(X)E(X^6)$$

$$\| \quad 0 \qquad\qquad 0$$

$$E(X^7) = \int_{-2}^{2} x^7 f_X(x)\,dx = \int_{-2}^{2} \frac{1}{4} x^7\,dx \qquad (>0)$$

$$f_X(x) = \begin{cases} \frac{1}{4} & \text{for } -2 \le x \le 2 \\ 0 & \text{else} \end{cases}$$