this time: double expectation theorem; utility;

next time: conditional variance; distributional review

AMS 131
23 May 19

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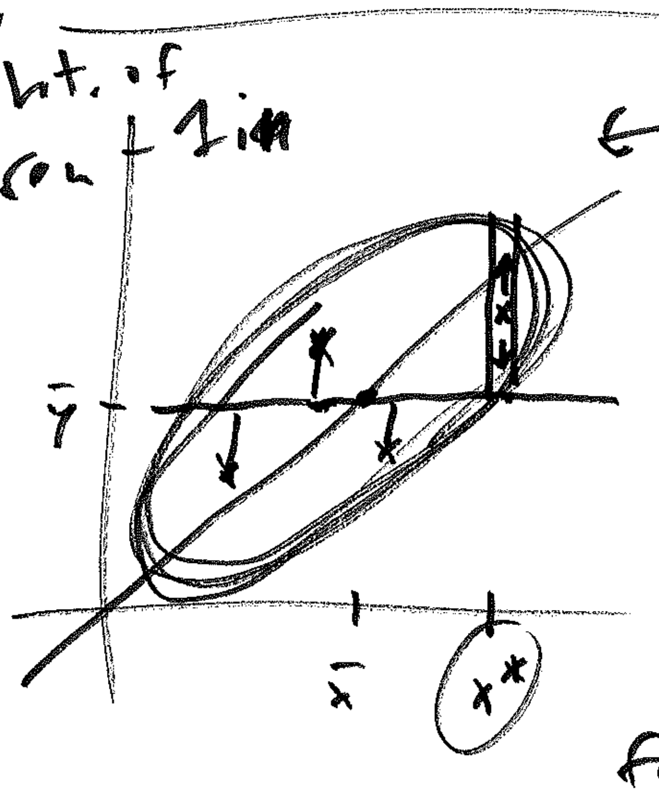


← Galton (1895)

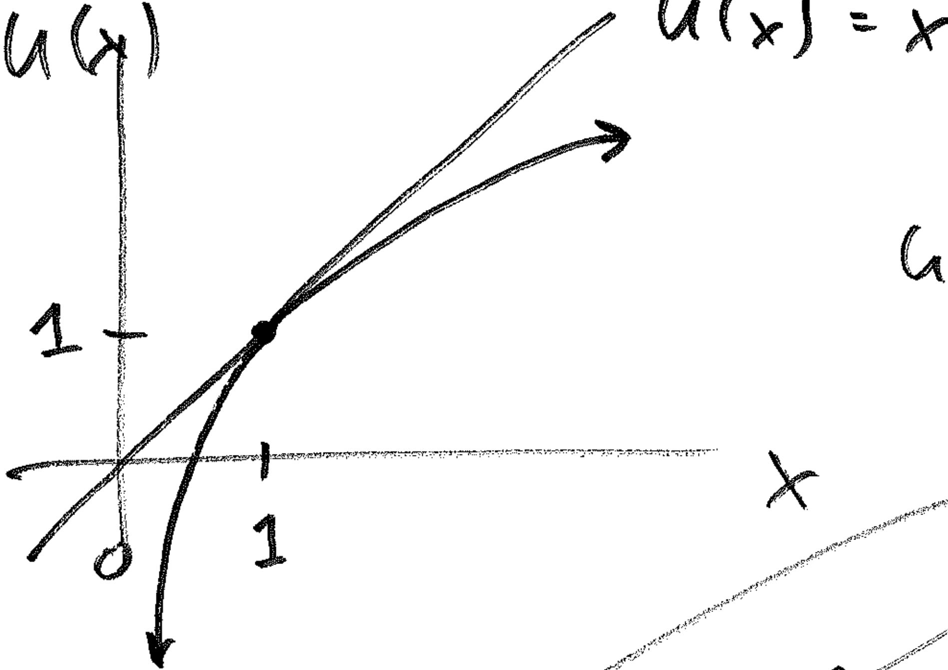
$$\left[\begin{array}{c|c} y_1 & x_1 \\ \vdots & \vdots \\ y_n & x_n \end{array}\right] \quad \updownarrow n$$

	y	x
mean	$\bar{y}$	$\bar{x}$
SD	s_y	s_x

correlation r

↑

K. Pearson (1890)

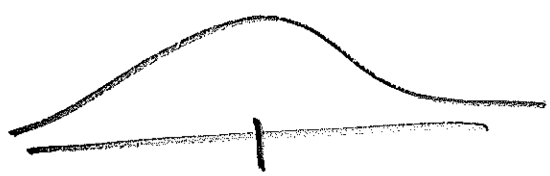


$u(x) = 1 + \log(x)$

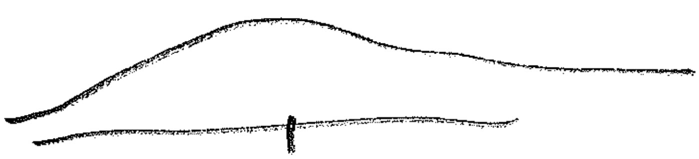
WLOG

u_r x ↑ u ↑

$u(\mathbb{X})$



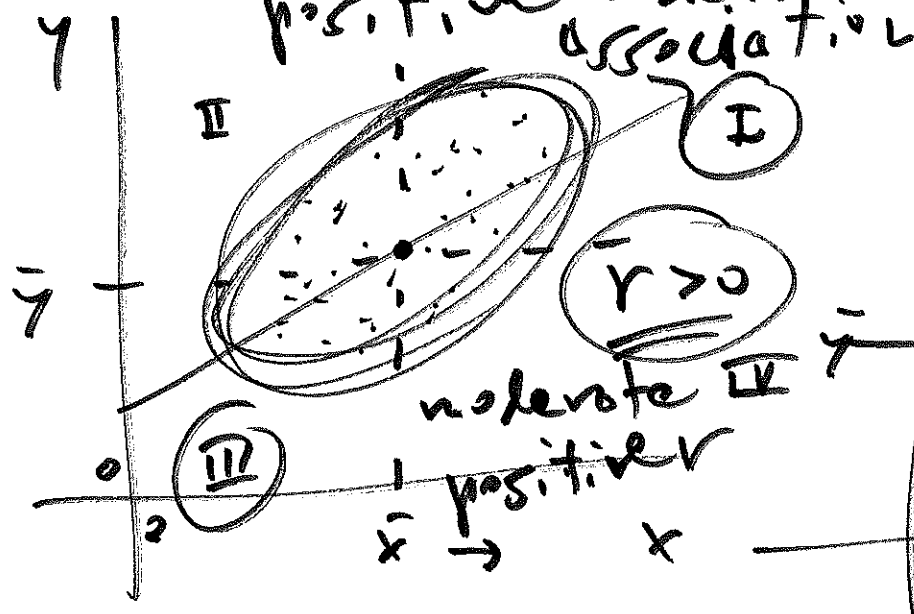
$u(\mathbb{X})$

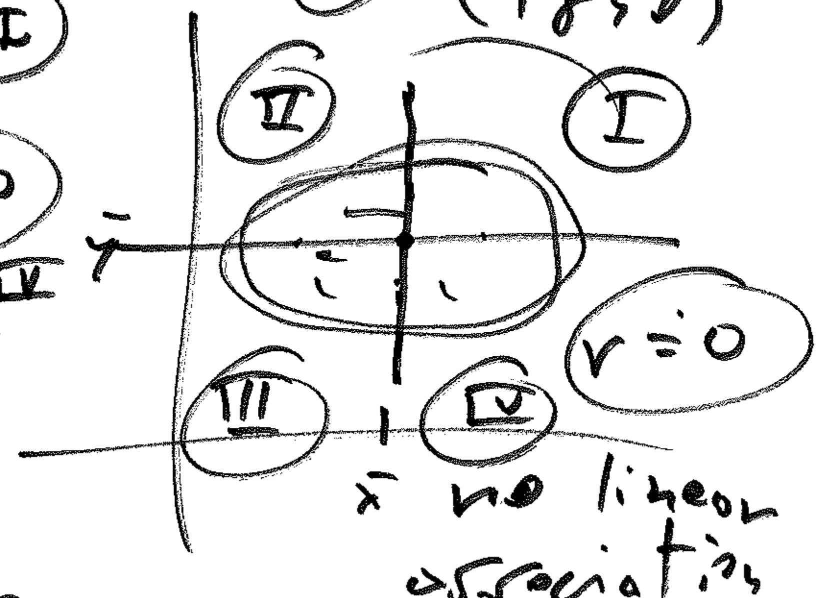


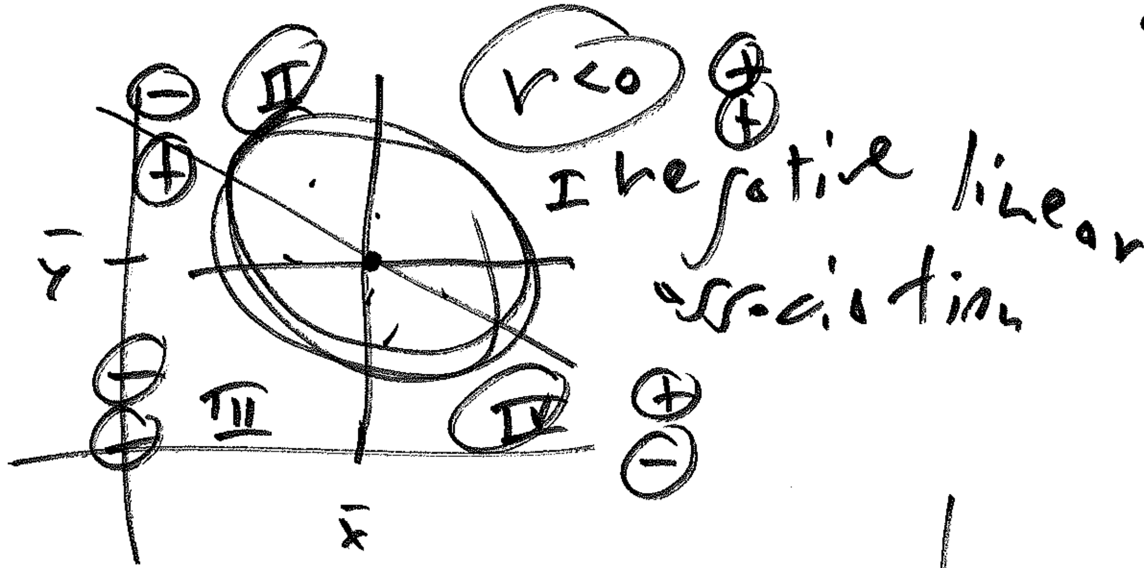
$$C(X, Y) = E[(X-\mu_X)(Y-\mu_Y)]$$

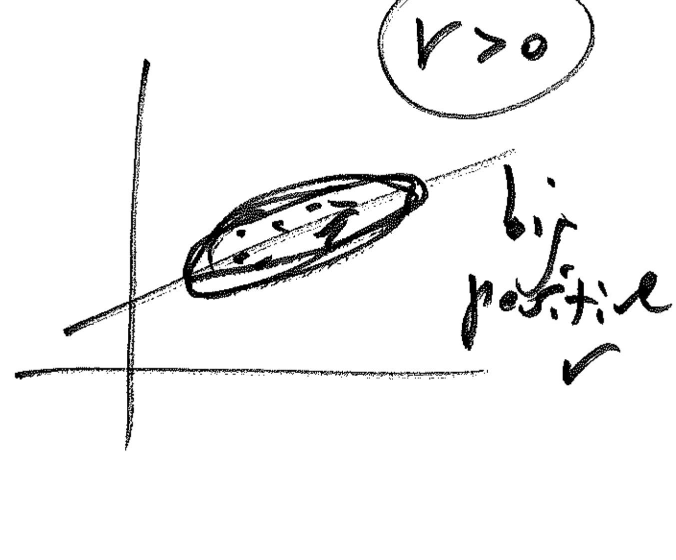
why!

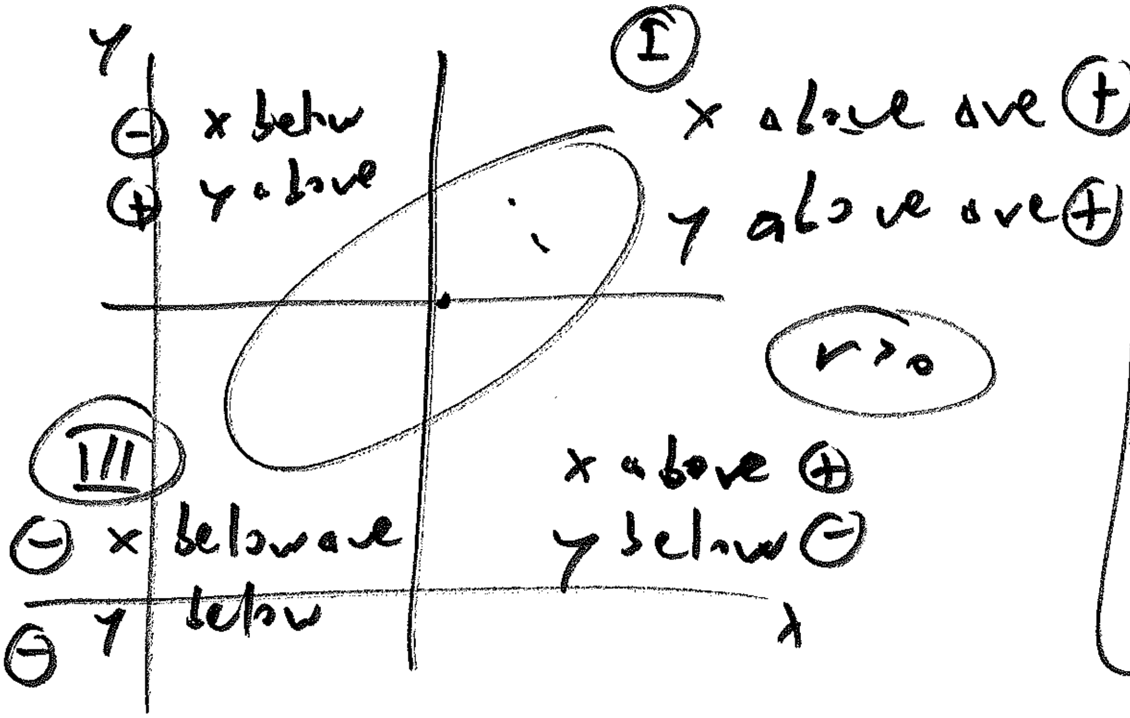
K. Pearson r (1890)

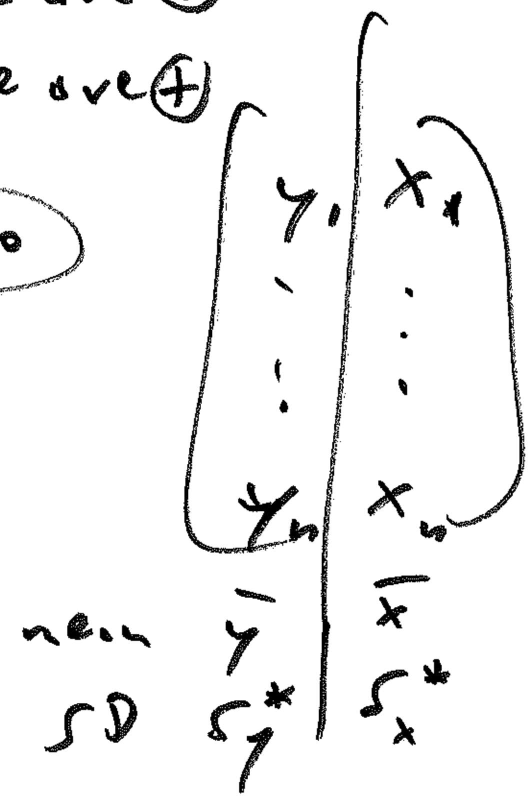












converting to standard units:

$$\frac{x_i - \bar{x}}{s_x}$$

data

$$\frac{X - E(X)}{SD(X)}$$

random variable

$$\boxed{\frac{1}{n}\sum_{i=1}^{n}\left(\frac{x_i - \bar{x}}{s_x^*}\right)\cdot\left(\frac{y_i - \bar{y}}{s_y^*}\right) = r}$$

sample cor.

$$-1 \le r \le +1$$

$$s_x^* = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

$$\rho = \frac{C(X, Y)}{SD(X) \cdot SD\, Y} = \frac{E\,(X - \mu_X)(Y - \mu_Y)}{SD(X) \cdot SD(Y)}$$

$$= E\left[\left(\frac{X - \mu_X}{SD(X)}\right) \cdot \left(\frac{Y - \mu_Y}{SD\, Y}\right)\right]$$

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pop

all British families with at least 1 son in 1895

ht. of son | ht. of f

$N = ?$ (big)

Y | X

mean μ_Y μ_X

SD σ_Y σ_X

corr. ρ

like IID →

sample

the observed families

ht. of son | ht. of f

y | x

$n = 1,000$

mean $\bar{y}$ $\bar{x}$

SD s_y s_x

corr r

$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$

$\mu_Y = E(Y)$