This is a case study.

Time: experiments;

Next events:

Time: sample space; set theory

Case Study 1: Tay-Sachs disease

Subject: "AMS 131" / "AMS 121"

Email

P(1 or more T-s in 5 children, both parents carriers) =?

True/False proposition

P(A) = set

Probability of

P(□) = 1 = 100% \( \frac{\text{A}}{\text{□}(1)} \)
\[ P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) \]

*Addition rule for OR*

\( P(A) \geq 0 \leq 1 \) or impossibility
certainty

\( A, B \) no overlap!

\( A, B \) mutually exclusive

\[ P(A \text{ or } (\text{not } A)) = 1 \]

\[ P(A) = 1 - P(\text{not } A) \]
\[ P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)} \]

- undefined if \( P(B) = 0 \)
- \( P(A \text{ and } B) = P(B) \cdot P(A \mid B) \)
- chain rule for \( \cap \)
- general product rule for \( \cap \)
- \( P(B \mid A) = \frac{P(B \text{ and } A)}{P(A)} = \frac{P(A \text{ and } B)}{P(A)} \)
- \( P(A \text{ and } B) = P(A) \cdot P(B \mid A) \)
\[ P(\overline{D}_1, G \text{ and } \overline{D}_2 = 9) = \]
\[ P(D_1 = 9) \cdot P(D_2 = 6 \mid D_1 = 9) \]
\[ = \frac{1}{3} \cdot 0 = 0 \quad \checkmark \]

\[ P((D_1 = 9 \text{ and } D_2 = 6) = \]
\[ P((D_1 = 9) \cdot P(D_2 = 6 \mid D_1 = 9) \]
\[ = P((D_1 = 9) \cdot P((D_2 = 6) \]

\[ \text{Bayesian} \]

if info about A doesn't change chances about B, it vice versa,
def A, B are independent
$$P(1 \text{ or more}) = P(\text{exactly 2+})$$

$$= P(\text{exactly 5}) = P(\text{exactly 7}) + \ldots + P(\text{exactly 7})$$

$$= 1 - P(0 + -5)$$

$$= 1 - \left( \prod_{i=0}^{5} \left( 1 - \frac{1}{4} \right) \right) \left( \prod_{i=2}^{5} \left( 1 - \frac{1}{4} \right) \right) \ldots \left( \prod_{i=2}^{5} \left( 1 - \frac{1}{4} \right) \right)$$
\[ P(1 \text{ or more } T-5) = 1 - (1 - \frac{1}{4})^5 \]

\[ = 76\% \]

(we just made a calculation with the binomial distribution)

\[ \begin{array}{c}
A \\
\hline
B \\
\hline
A \cup B
\end{array} \]

\[ \begin{array}{c}
A \\
\hline
B \\
\hline
A \cap B
\end{array} \]

Pascal's triangle:

\[
\begin{array}{cccccc}
& & & & & 1 \\
& & & 1 & & \\
& & 1 & 2 & 1 & \\
& 1 & 3 & 3 & 1 \\
1 & 4 & 6 & 4 & 1
\end{array}
\]