AMS 131
4 Apr 19

this case study;
time: experiments;
next time: events;
sample space; set theory

read: DS ch. 1

office hours set this quarter (1)
starting on Fri 5 Apr 19
(see course webpage)

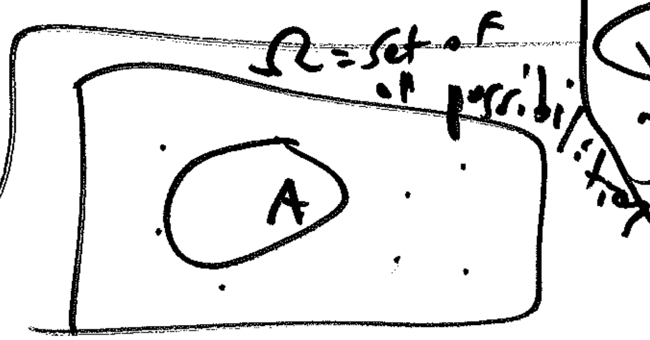
Case Study 1: Tay-Sachs (T-S) Disease

subject: email | "AMS_131" / "AMS131"

P(1 or more T-S in 5 children, both parents carriers) = ?

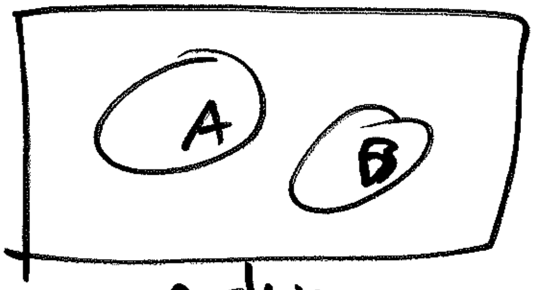
P(A) ← true/false proposition
A ← set
P ← probability of

John Venn Diagram



P(□) = 1 = 100%

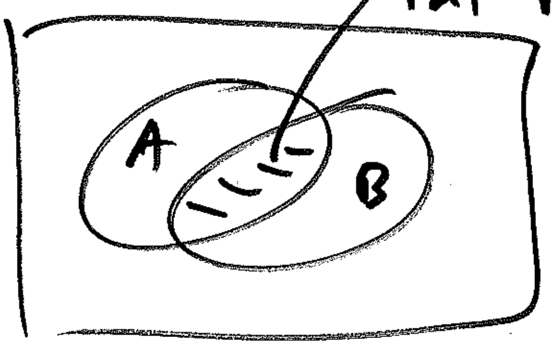
$$P(A) = \frac{A}{\Omega} \quad (1)$$



$P(A \text{ or } B) =$

$P(A) + P(B)$

no overlap
$P(A \text{ and } B) = 0$



intersection
(overlap)
(A <u>and</u> B)

$P(A \text{ or } B) =$

$P(A) + P(B)$

$- P(A \text{ and } B)$

addition rule for $\boxed{\text{OR}}$

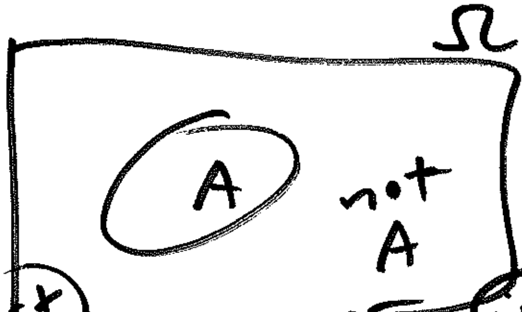
$0 \le P(A) \le 1$

impossibility

certainty

A, B no overlap:

A, B mutually exclusive



$P(A \text{ or } (\text{not } A)) = 1$

$P(A) + P(\text{not } A)$

direct

indirect

$P(A) = 1 - P(\text{not } A)$

$P(A \text{ and } B) \stackrel{?}{=} P(A) \overset{?}{\cdot} P(B)$

2 cases | IID, SRS — sampling

population

$N=3 \begin{bmatrix} 1 \\ 2 \\ 9 \end{bmatrix} \xrightarrow{\text{at random}} \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} \quad n=2$

$P(Y_1 = 9 \text{ and } Y_2 = 9) = ?$

IID (at random with replacement)

2nd draw (Y_2)

Y_1 1st draw	1	2	9
1	(1,1)	(1,2)	(1,9)
2	(2,1)	(2,2)	(2,9)
9	(9,1)	(9,2)	(9,9)

$\boxed{\text{ELM?}}$ yes

(9 elemental outcomes (EOs))

$P(Y_1 = 9 \text{ and } Y_2 = 9) = \frac{1}{9}$

$P(Y_1 = 9) = \frac{1}{3} = \frac{3}{9}$ | $P(Y_2 = 9) = \frac{1}{3} = \frac{3}{9}$ ✓

$P(Y_1 = 9 \text{ and } Y_2 = 9) = \frac{1}{9} = P(Y_1 = 9) \cdot P(Y_2 = 9)$

SRS (at random without replacement)

Y_2 (2nd draw)

Y_1 (1st draw)	1	2	9
1	$\cancel{(1,1)}$	(1,2)	(1,9)
2	(2,1)	$\cancel{(2,2)}$	(2,9)
9	(9,1)	(9,2)	$\cancel{(9,9)}$

$\boxed{\text{ELM?}}$ yes

$P(Y_1 = 9 \text{ and } Y_2 = 9) = 0$

$P(Y_1 = 9) = \frac{1}{3} = \frac{2}{6}$

$P(Y_2 = 9) = \frac{1}{3} = \frac{2}{6}$

$P(Y_1 = 9 \text{ and } Y_2 = 9) = 0 \neq \frac{1}{3} \cdot \frac{1}{3}$

$P(Y_1 = 9) \cdot P(Y_2 = 9)$

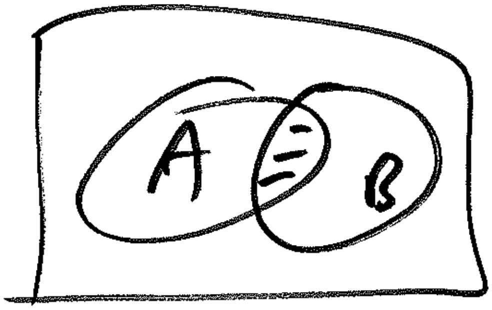
Abraham de Moivre (1705)

Thomas Bayes (1760)

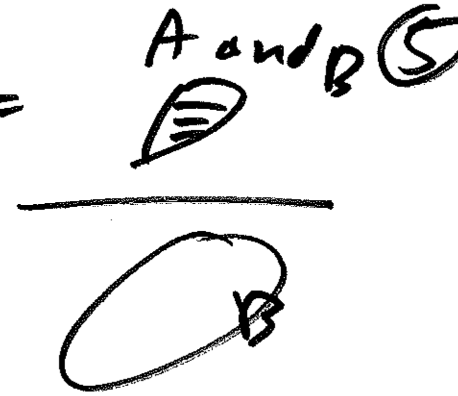
conditional probability

$$P(A) = \frac{A}{\boxed{}} \quad (1)$$

$\boxed{A}$



$P(A \mid B) =$



"given"

$$P(A \mid B) = \begin{cases} \dfrac{P(A \text{ and } B)}{P(B)} & \text{if } P(B) > 0 \\ \text{undefined} & P(B) = 0 \end{cases}$$

~~general product rule for and~~

$$P(A \text{ and } B) = P(B) \cdot P(A \mid B)$$

$$= P(A) \cdot P(B \mid A)$$

chain rule for and

$$P(B \mid A) = \frac{P(B \text{ and } A)}{P(A)} = \frac{P(A \text{ and } B)}{P(A)}$$

$$P(A \text{ and } B) = P(A) \cdot P(B \mid A)$$

$$P(X_1 = 9 \text{ and } X_2 = 9) =$$

$$P(X_1 = 9) \cdot P(X_2 = 9 \mid X_1 = 9)$$

$$= \frac{1}{3} \cdot 0 = 0 \checkmark$$

$$P_{IID}(X_1 = 9 \text{ and } X_2 = 9) =$$

$$P_{IID}(X_1 = 9) \cdot P_{IID}(X_2 = 9 \mid \underline{X_1 = 9})$$

$$= P_{IID}(X_1 = 9) \cdot P_{IID}(X_2 = 9)$$

$\boxed{\text{Bayesian}}$

if info about A doesn't change chances about B, & vice versa,

$\boxed{\text{def.}}$ A, B are <u>independent</u>

frequentist definition

(if and only if)

A, B independent $\overset{\text{iff}}{\longleftrightarrow}$ $P(A \text{ and } B) = P(A) \cdot P(B)$

I I D

independent

identically distributed

T-5 | P(1 or more T-5 babies)

# of T-5 babies

if ELM applies $P(\text{T-5 from } \uparrow) = \frac{5}{6}$

but ELM does not apply

0
1
2
3
4
5

6 "EOs" $P(1) > P(0)$

bogus

$P($1 or more T-s$) = P($exactly 2 Ts

or ... or exactly 5$) =$

$P($exactly 1$) + \cdots + P($exactly 5$)$

$P($1 or more T-s$) =$

$1 - P($0 T-s$)$

$= 1 - P($not T-s on 1st and not T-s on 2nd and ... and not T-s on 5th$)$

biology
$\downarrow$
I ID

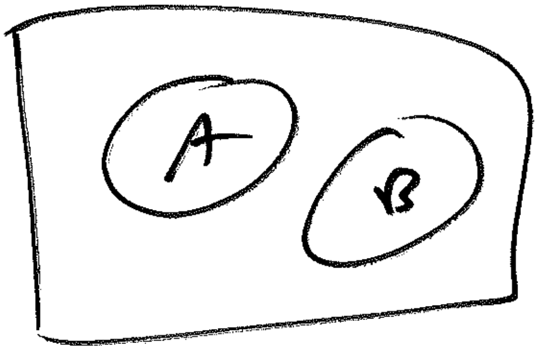
$= 1 - P($not T-s on 1st$) \cdot P($not T-s on 2nd$) \cdots P($not T-s on 5th$)$

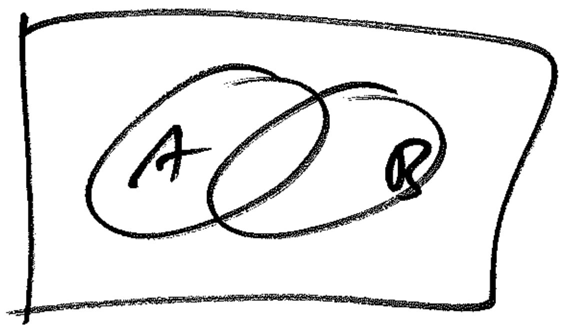
biology
ID

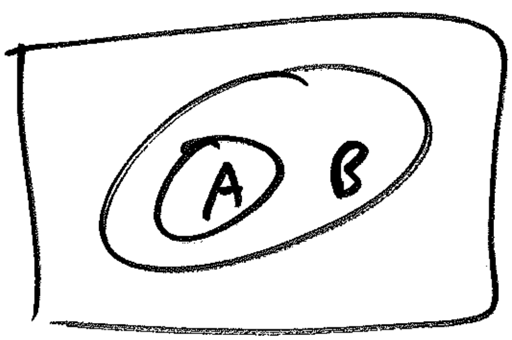
$= 1 - \left(1 - \frac{1}{4}\right) \cdot \left(1 - \frac{1}{4}\right) \cdots \left(1 - \frac{1}{4}\right)$

$$P(1 \text{ or more T-s}) = 1 - \left(1 - \frac{1}{4}\right)^5 = 76\%$$

(we just made a calculation with the **Binomial** distribution







Pascal's triangle

				1				
			1		1			
		1		2		1		
	1		3		3		1	
1		4		6		4		1