From Kolmogorov's axioms, \( P(A) = 0 \) implies that the subset \( (\forall A_i) \) is disjoint, as follows from \( \text{Kolmogorov's inequality} \).

If \( A_i \) is disjoint, then the finite number of disjoint events is a small measure.

But it is hard to motivate: it's a small measure, which is not just for a piece of genius on Kolmogorov's part but also for a countable collection (Kolmogorov) turns out to be absolutely necessary.
2. \( P(A^c) = 1 - P(A) \)

3. If \( A \subseteq B \) then \( P(A) \leq P(B) \)

4. For all events \( A \),
   \[ 0 \leq P(A) \leq 1 \] (the axiomatic rule)

5. For all events \( A, B \),
   \[ P(A \cup B) = P(A) + P(B) - P(A \cap B) \]
   or
   \[ \text{general addition rule for or} \]

6. (attributed to the Italian mathematician, Carlo Bonferroni (1892-1960)): For any events \( A_1, A_2, \ldots, A_n \),
   \[ P(\bigcup_{i=1}^{n} A_i) \leq \sum_{i=1}^{n} P(A_i) \] and
   \[ P(\bigcap_{i=1}^{n} A_i^c) \geq 1 - \sum_{i=1}^{n} P(A_i) \]
   useful in statistics
Tay-Sachs disease is more detailed...

<table>
<thead>
<tr>
<th>NNNNN</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>TNNNN</td>
<td>1</td>
</tr>
<tr>
<td>TTTTT</td>
<td>5</td>
</tr>
<tr>
<td>TTTTN</td>
<td>2</td>
</tr>
<tr>
<td>TTTTN</td>
<td>3</td>
</tr>
<tr>
<td>TTTTN</td>
<td>4</td>
</tr>
<tr>
<td>TTTTN</td>
<td>5</td>
</tr>
</tbody>
</table>

The number of T→S alleles = 5 left

see if we can work out

\[ P(T = 1), \ P(T = 2), \ldots \]

\[ P(T = 5); \text{ we already worked on} \]

\[ P(T = 0) = \ P(\text{exactly } 0 \ T\rightarrow S) \]

\[ = \ \left( \left( \frac{2}{5^T} \right) \ \text{not} \right) \ \left( \frac{2}{5^T} \right) \ \text{not} \ \left( \frac{5}{T-S} > \right) \]

\[ \text{independence} \]

\[ = \ \ P(\text{not}) \ \left( \frac{2}{5^T} \right) \ \text{not} \ \left( \frac{5}{T-S} \right) \]

\[ \text{identical distribution} \]

\[ \left[ 1 - P(\frac{9}{5^T}) \right] \ldots \]

\[ \left[ 1 - P(\frac{9}{5^T}) \right] = 1 \ P(1-p)^5 \ \text{with} \]

\[ 0.5 \ P = \frac{1}{4} \]
A similar line of reasoning gives 

\[ P(I = 5) = P(TTTTT) = p^5 = \frac{1}{10} p^5 (1-p)^5. \]

What about \( P(I = 1) \)? The table on the previous page lists all of the outcomes with 1 T's only: they all have 1 T and 4 N's, so each one has probability \( p (1-p)^4 \), and there are 5 of them, so \( P(I = 1) = 5 p (1-p)^4 \).

By similar reasoning \( P(I = 2) = 10 p^2 (1-p)^3 \).

The outcomes with \( I = 3 \) are mirror images of those with \( I = 2 \): \( \{TTNNNN\} \) \( \{NNTTTT\} \).
So there must also be 10 elements of \( S \) with \( \tilde{X} = 3 \) and \( p(\tilde{X} = 3) = 10 \cdot p^3(1-p)^2 \).

And finally, \( \tilde{X} = 4 \) is a minor image of \( \tilde{X} = 1 \) so \( p(\tilde{X} = 4) = 5 \cdot p^4(1-p)^1 \).

<table>
<thead>
<tr>
<th># of T-S babies</th>
<th>( p(\tilde{X} = y) )</th>
<th>( y ) with ( p = \frac{1}{4} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>( \frac{1}{5}(1-p)^5 )</td>
<td>0.2373</td>
</tr>
<tr>
<td>1</td>
<td>( 5p^1(1-p)^4 )</td>
<td>0.3955</td>
</tr>
<tr>
<td>2</td>
<td>( 10p^2(1-p)^3 )</td>
<td>0.2637</td>
</tr>
<tr>
<td>3</td>
<td>( 10p^3(1-p)^2 )</td>
<td>0.0879</td>
</tr>
<tr>
<td>4</td>
<td>( 5p^4(1-p)^1 )</td>
<td>0.0146</td>
</tr>
<tr>
<td>5</td>
<td>( 1p^5(1-p)^0 )</td>
<td>0.0010</td>
</tr>
</tbody>
</table>

Soon we'll call \( \tilde{X} \) a random variable (symbolizing the data generating process) and never use \( y \) to stand for a possible value of \( \tilde{X} \).
So it looks like
\[ P(Y = y) = \binom{n}{y} p^y (1-p)^{n-y}. \]

We could even be a bit more symbolic and note that \( n = 5 \) is the number of times the basic dichotomy (T vs. N) occurs in this case study, so
\[ P(Y = y) = \binom{5}{y} p^y (1-p)^{5-y}. \]

What about \( \square \)? You can see that the multipliers \( \square \) come from Pascal’s Triangle, but can we write down a formula for them?

Example: You have an ordinary deck of \( n = 52 \) playing cards.
How many possible poker hands of \( k = 5 \) cards can you draw at random without replacement from the deck?

It's like filling in 5 slots: \( \_ \_ \_ \_ \_ \)

The first slot can be filled in \( n = 52 \) ways, and the second in \( (n-1) = 51 \) ways, ..., the 5th slot in \( (n-k+1) = 48 \) ways; so the total # of ways you can do this is \( 52 \cdot 51 \cdot 50 \cdot 49 \cdot 48 = n(n-1) \cdots (n-k+1) = 311,875,200 \) ways. This is called the number of permutations of 52 things taken 5 at a time.
Definition \[P_n^k = n(n-1) \cdots (n-k+1).\]

How many possible orderings of a 52-card deck are there? Now there are 52 slots, e.g., \(3\) of \(\bullet\) \( \cdots \) of \(\bullet\), so the total number must be \(52 \cdot 51 \cdots 1 = \text{Def.} \]

\[n(n-1) \cdots 1 = n! \text{ read } n \text{ factorial}\]

80658175170943878571660636856403766975289
5054408832178240000000000000000 = 8.1.10^{67}

Free from alpha (Maple (Apr 19)