Definition: An experiment $E$ is a data-generating process in which all possible outcomes can be listed before $E$ is performed.

Definition: An event $E$ is a set of possible outcomes of an experiment $E$.

Example: Tay-Sachs disease $E$ = (the process by which the husband & wife end up with 5 children, each a T-S baby or not) \[ \text{the E of interest is } E = \{ \text{at least 1 T-S baby} \} \]
Definition: The simple space \( \mathcal{S}(E) \) is the set of all possible outcomes of an experiment \( E \).

Example: \( \{T-5\} \)

Let \( T = \{T-5 \text{ baby}\} \) and \( N = \{\text{not T-5 baby}\} \).

Here \( \mathcal{S} = \{\text{NNNNN}, \ldots, \text{TFFFFF}\} \).

Since there are 2 possibilities for each baby \( (T, N) \) and 5 babies, the number of elements in \( \mathcal{S} \) is \( 2^5 = 32 \).

\( \mathcal{S} \) is an example of a product space:

\[ \{T, N\} \times \{T, N\} \times \ldots \times \{T, N\} = \{T, N\}^5. \]
Here \( E = \{ \text{TNNNN}, \ldots, \text{TTTTT} \} \).

**Notation** Use \( s \) to stand for the individual outcomes (elements) of \( S \).

The theory of probability we'll look at in this class was developed by Kolmogorov (1933) in an attempt to rigorize the hypothetical process of throwing a dart at a

\[ s \in S \]

Venn diagram (rectangle)

The rules of this dart-throwing were simple: 0) the dart must land somewhere inside (or on the boundary of) the rectangle \( S \), which
Symbolically stands for the sample space, and all the points where the dart might land in \( S \) are "equally likely" (as yet, an undefined concept).

**Definition**

The complement \( A^c \) of a set \( A \) in \( S \) is the set that contains all elements of \( S \) not in \( A \)

(You can see from the Venn diagram on p. 6 that the dart has to fall either in \( A \) or in \( A^c \), which we could also call \( \text{not } A \).)\n
\[ \in \] \( s \in S \) means that \{ outcome \} \( s \) belongs to \( S \).
Definition | A set $A$ is contained in another set $B$ (write $A \subseteq B$) if every element of $A$ is also in $B$; we can also say that $B$ contains $A$ ($B \supseteq A$).

Evidently, if $A$ and $B$ are events, $A \subseteq B$ (iff) (if and only if) if $A$ occurs then so does $B$.

(Thm) Consequences | If $A$, $B$, $C$ are events then (a) $A \subseteq B$ and $B \subseteq C \iff A = B$ and (b) $A \subseteq B$ and $B \subseteq C \implies A \subseteq C$.

Definition | The cardinality of a set $A$ (written $|A|$) is the number of distinct elements in $A$. 
Example (Tay-Sachs) \( |S'| = 32 \) (see 102)

**Definition** The set of all subsets of a given set \( S \) is called the **power set** of \( S \), denoted by \( 2^S \); this notation was chosen because, if \( |S| = n \), then \( |2^S| = 2^n \) (in other words, if \( S \) has \( n \) distinct elements then there are \( 2^n \) distinct subsets of \( S \).

**Definition** It's convenient to have a symbol for the set that has no elements in it: \( \emptyset \), the **empty set**.
Example: If $S = \{a, b, c\}$ then $|S| = 3$ and the power set has $2^3 = 8$ sets in it. Given any set $S$, Kolmogorov (1933) wanted to be able to define probabilities in a logically-consistent manner (in other words, free from contradictions or paradoxes) to all of the sets in $2^S$. If $|S|$ is finite, it turns out that nothing nasty can happen.
But if \( |S| \) is infinite, nasty things can unfortunately happen. [Definite]

A set with an infinite number of distinct elements is called an infinite set.

(4 Apr 19)

**Definition** If the elements of an infinite set \( A \) can be placed in 1-to-1 correspondence with the positive integers \( N = \{ 1, 2, 3, \ldots \} \), \( A \) is said to be countably infinite.

**Example** The rational numbers are those real numbers that can be expressed as ratios of integers (ex. \( \frac{1}{2}, \frac{14}{13}, -\frac{89}{212} \)...)