AMS 131 extra notes 1

Definition An *experiment* $\mathcal{E}$ is a data-generating process in which all possible outcomes can be listed before $\mathcal{E}$ is performed.

Definition An *event* E is a set of possible outcomes of an experiment $\mathcal{E}$.

Example Tay-Sachs (T-S) disease

$\mathcal{E}$ = (the process by which the husband & wife end up with 5 children, each a T-S baby or not). The E of interest is $E = \{$at least 1 T-S baby$\}$

Definition The sample space $S^{(\Omega)}$ is the set of all possible outcomes (elements) of an experiment $\mathcal{E}$.

Example: T-S

Let T = (T-S baby) and N = (not T-S baby)

NNNNN

TNNNN
NTNNN
NNTNN
NNNTN
NNNNT
TTNNN
TNTNN
⋮
TTTTT

E

Here $S = \{NNNNN, \ldots, TTTTT\}$

Since there are 2 possibilities for each baby (T, N) and 5 babies, the number of elements in S is $2^5 = 32$.

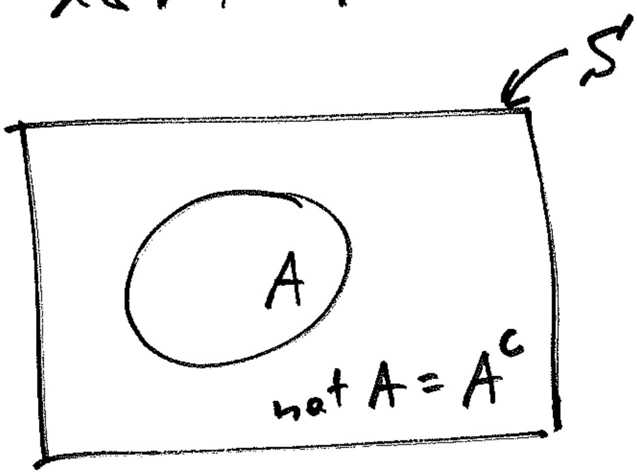
S is an example of a product space:

$$\underbrace{\{T,N\} \times \{T,N\} \times \cdots \times \{T,N\}}_{5} = \{T,N\}^5.$$

Here $E = \{TNNNN, \ldots, TTTTT\}$.

Notation Let's use s to stand for the individual outcomes (elements) of S.

The theory of probability we'll look at in this class was developed by Kolmogorov (1933) in an attempt to rigorize the hypothetical process of repeatedly throwing a dart at a Venn diagram (rectangle)



The rules of this dart-throwing were simple: ① the dart must land somewhere inside (or on the boundary of) the rectangle S, which

symbolically stands for the sample space, (4)
and ② all the points where the dart might land in S are "equally likely" (as yet, an undefined primitive concept).

$\boxed{\text{Definition}}$ The complement A^c of a set A contained in S is the set that contains all elements of S not in A

(You can see from the Venn diagram on p. ③ that the dart has to fall either in A or in A^c, which we could also call $\boxed{\text{not } A}$.) $\boxed{\text{Notation}}$

$s \in S$ means that $\left\{ \begin{matrix} \text{outcome} \\ \text{element} \end{matrix} \right\}$ s belongs to S
$\in$ is an element of

Definition A set A is <u>contained in</u> another set B (A is a <u>subset</u> of B) (written $A \subset B$) if every element of A is also in B; can also say that B <u>contains</u> A ($B \supset A$).

Evidently, if A and B are events,

$A \subset B \overset{(\text{iff})}{\longleftrightarrow}$ (if and only if) if A occurs then so does B

Consequences (theorem) If A, B, C are events

then (a) $A \subset B$ and $B \subset A \leftrightarrow A = B$

and (b) $A \subset B$ and $B \subset C \rightarrow A \subset C$.

Definition The <u>cardinality</u> of a set A (written $|A|$) is the number of distinct elements in A.

Example (Tay-Sachs) | $|S| = 32$ (see p. 2)

Definition The set of all subsets of a given set S is called the <u>power set</u> of S, denoted by 2^S; this notation was chosen because, if $|S| = n$, then $|2^S| = 2^n$ (in other words, if S has n distinct elements then there are 2^n distinct subsets of S.

Definition It's convenient to have a symbol for the set that has no elements in it: $\emptyset$, the <u>empty</u> set.

Example: If $S = \{a, b, c\}$ then

$|S| = 3$ and the power set has $2^3 = 8$

sets in it.

$\emptyset$	1
$\{a\}$	
$\{b\}$	3
$\{c\}$	
$\{a, b\}$	
$\{a, c\}$	3
$\{b, c\}$	
$\{a, b, c\} = S$	1

Given any set (sample space) S, Kolmogorov (1933) wanted to be able to define probabilities in a logically-internally-consistent manner (in other words, free from contradictions or paradoxes) to all of the sets in 2^S.

$$\begin{array}{ccccccc} & & & 1 & & & \\ & & 1 & & 1 & & \\ & 1 & & 2 & & 1 & \\ 1 & & 3 & & 3 & & 1 \end{array}$$

Pascal's triangle

If $|S|$ is finite, it turns out that nothing nasty can happen.

But if $|S|$ is infinite, nasty things can unfortunately happen.

Definition

A set with an infinite number of distinct elements is called an infinite set.

(4 Apr 19)

Definition | If the elements of an infinite set A can be placed in 1-to-1 correspondence with the positive integers $\mathbb{N} = \{1, 2, 3, \dots\}$, A is said to be countably infinite.

Example | The rational numbers are those real numbers that can be expressed as ratios of integers (ex. $\frac{1}{2}, \frac{14}{13}, -\frac{89}{2121}, \dots$)