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AMS 131: Quiz 8

Name: _____

Someone offers you the possibility to play a gambling game with the following rules. First, you decide how much money you're willing to put at risk in this game: this amount — let's call it A — is referred to as your *stake* (all the monetary quantities are in dollars in this problem). Having chosen your stake, you're allowed to bet any amount $0 \leq B \leq A$ (thus, as a decision problem, for any fixed value of A, your possible actions in this situation correspond to values of B). If you win the bet, which occurs with probability $0 < p < 1$, your stake becomes $(A + B)$; if you lose, it becomes $(A - B)$, and this (of course) occurs with probability $(1 - p)$; and (crucially) p is known to you. Let X denote the value of your stake after the gamble has occurred, and suppose that you agree with Daniel Bernoulli that a reasonable utility function is $U(x) = 1 + \log(x)$.

(a) Write out the probability mass function (PMF) for X.

(b) Work out your expected utility $E[U(X)]$ in this game, as a function of A, B and p.

(c) Intuitively, what should you do (i.e., what value of B should you choose) if $p < \frac{1}{2}$? Explain briefly.

(d) Show that when $p \geq \frac{1}{2}$ your expected utility is maximized with the choice $B = (2\,p - 1)\,A$. Is this answer intuitively reasonable? Explain briefly.